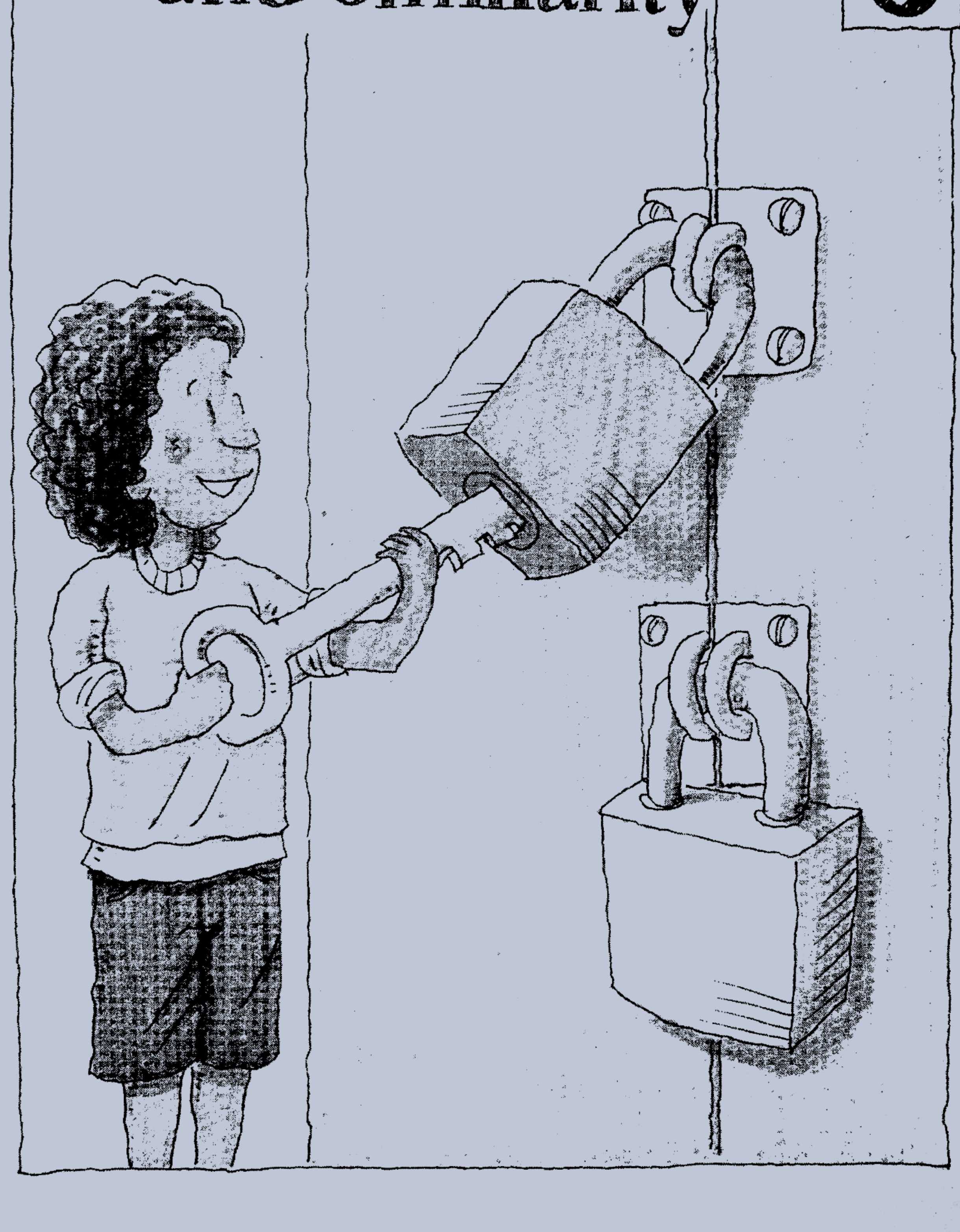
6

Transformations and Similarity



CHAPTER 6 Transformations and Similarity

You may not often take the time to think about how you move objects around as you go about daily life. Many of the movements you make in a given day involve changing directions. In Section 6.1, you will investigate different kinds of motion on a coordinate graph. You will learn how to give directions to slide, flip, turn, and stretch flat shapes. You will also learn how to show where the shapes will be after a series of moves.

In Section 6.2, you will use mathematics you already know to investigate how shapes change size. You will also determine the unknown length of a side in a figure with given information about the lengths of other sides in the figure and in related figures.

Guiding Questions

Think about these questions throughout this chapter:

How can I visualize it?

How can I describe the motion?

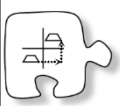
How can I transform it?

How do they compare?

What is the relationship?

In this chapter, you will learn:

- Transform shapes by flipping, turning, and sliding them on a coordinate graph.
- Describe movement on a graph using coordinates and expressions.
- Compare shapes and use similarity to find missing side lengths of polygons, especially triangles.



Section 6.1 You will use an eTool to move a shape on a coordinate graph using slides, flips, and turns, and will use integers to describe those moves.

Section 6.2 This section will introduce similarity and congruence for polygons.

Lesson 6.1.1 RIGID Transformations

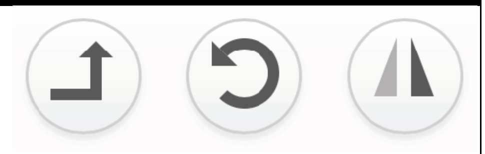
How can I move a shape on a grid?

How can you describe the movement of a figure on a flat surface when it is not moving in a straight line? For example, when you need to move a loose puzzle piece into the puzzle how can you describe the way its position changes?

Today you will explore mathematical ways of sliding, turning, and flipping an object without changing its size or shape. These types of movements are called **rigid transformations**. You will solve various puzzles as you explore the transformations.

Problem 6-1

KEY-IN-THE-LOCK PUZZLES



Are you ready for a puzzle challenge? You will use the *Key-Lock Puzzle* (CPM). Your job will be to move the key to the keyhole to unlock the door, using the transformation buttons shown at right.

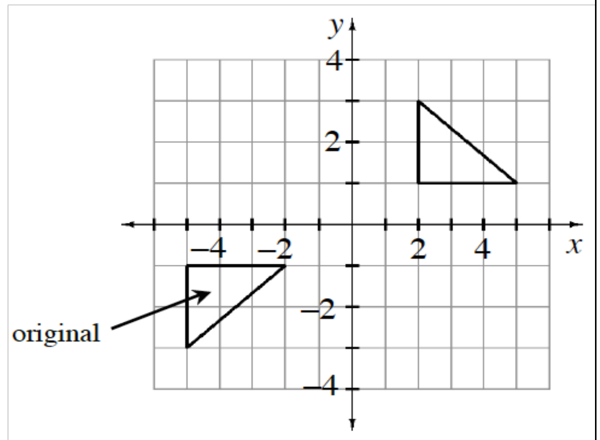
You will need to tell the computer about how you want the key to move. For example, how far to the left or right and how far up or down do you want the key to slide? In which direction do you want your key to flip?

Your Task: For each puzzle, move the key to the keyhole. Remember that to unlock the door, the key must fit exactly into the keyhole. Also note that your key will not be able to move through walls. There are 12 puzzles to complete. Go to the ebook and choose a puzzle by clicking the Paper/Pencil icon and selecting a puzzle or select the puzzles directly below.

Problem 6-1	**Transformations Record Sheet**	
Challenge	**Moves**	**Details**
Example	Slide Turn	Horizontal: +10 units and Vertical: +2 units. Clockwise 90° about the tip of the key.
1		
2		
3		

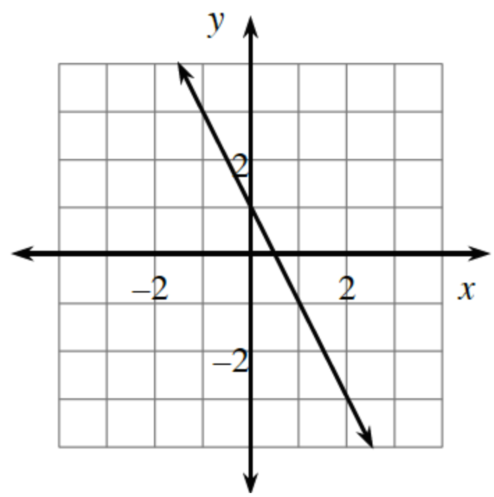
Review & Preview	Problem 6-2

Describe what moves you could use to create the transformation of the original image shown below.



Problem 6-3

Review what you know about graphs as you complete parts (a) through (d) below.



a. Find the equation of the line graphed at right.

b. What are the x- and y-intercepts?

c. On the same set of axes, graph a line that is *parallel* to the line graphed at right and that goes through the *origin* (0,0). Find the equation of this new line.

Problem 6-4

Which equation below has *no* solution? Explain how you know.

a. $4(x+1) = 2x + 4$

b. $9 - 5x + 2 = 4 - 5x$

Problem 6-5

Rena says that if ***x=−5,*** the equation below is true. Her friend, Dean, says the answer is ***x=3***. Who is correct? Justify your conclusion.

$$9(x+4) = 1 + 2x$$

Problem 6-6

Find the rule for the pattern represented below.

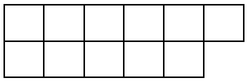
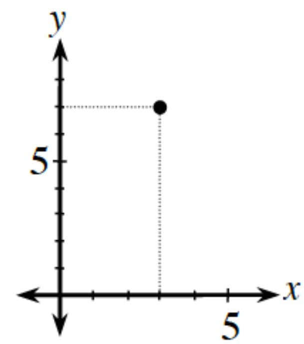


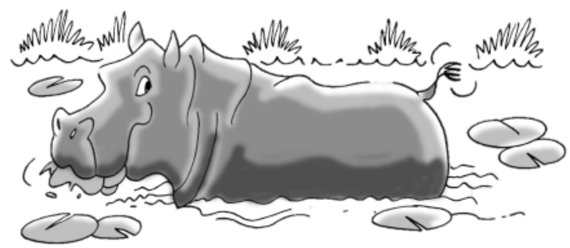
Figure 1



Problem 6-7

Homer the Hungry Hippo is munching on the lily pads in his pond. When he got to the pond, there were 30 lily pads, but he is eating 5 lily pads an hour. Henrietta the Hungrier Hippo found a better pond with 38 lily pads! She eats 7 lily pads every hour.

Define the variables and write and solve a system of equations.



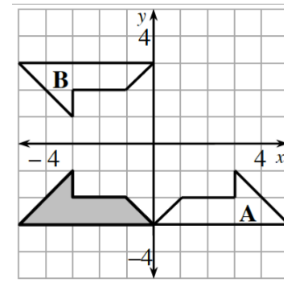
a. If Homer and Henrietta start eating at the same time, when will their ponds have the same number of lily pads remaining?

b. How many lily pads will be left in each pond at that time?

Lesson 6.1.2
RIGID Transformations on a Coordinate graph

How can I move the shape on the grid?

Have you ever had trouble giving directions? Sometimes describing where something is or how it has moved is difficult. For this reason, people often use coordinate graphs like the one shown at right. Coordinate graphs help you describe directions with words like "left" and "down." They can also help you measure distances.

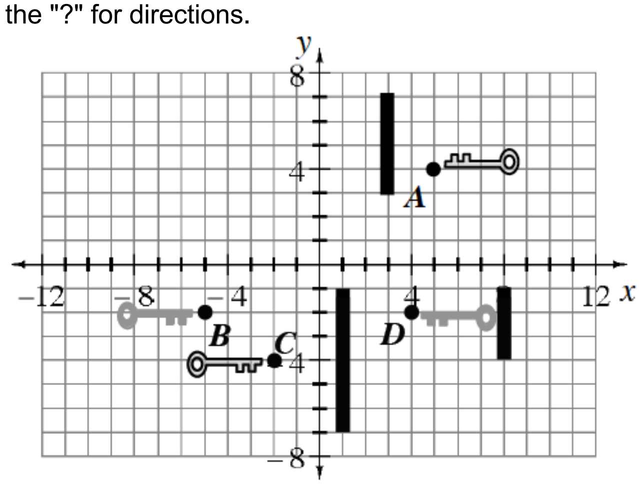


Today you will work with your team to describe movement on a coordinate graph. You will also look at ways to describe where an object is on the grid before and after a transformation. As you work, use the questions below to help start math discussions with your team members.

Is there a different way to get the same result? Did we give enough information?
How can we describe the position?

Problem 6-8

While solving the key challenge in Lesson 6.1.1, Rowan made more than one move to change his key from point A to point B and from point C to point D, as shown on the graph at right. Both of these keys are shown as triangles on the Lesson 6.1.2 Resource Page. Explore using the *Challenge 1* and *Challenge 2* puzzles (etool). You may need to resize the window and reposition the grid by shift dragging. Refer to the "?" for directions.



Your Task: With your team, describe how Rowan could have moved each key from the starting position to the ending position using slides (also called **translations**), turns (also called **rotations**), and/or flips (also called **reflections**).

- Make sure you provide enough detail to describe the moves completely.
- Try to find more than one way he could have moved each key.
- Be ready to justify your ideas with the class.

A to B	C to D

Problem 6-9

WHERE DOES IT LAND?

Felicia found a copy of a puzzle like the one in problem 6-1, but the lock is missing. All she has are the starting points and the moves to unlock the lock. This time her key is shaped like a triangle.

The points are at A(−5,0), B(−5,3), ***and*** C(−1,0)***.***

Step 1: ***Translate*** 4 ***units to the right and*** 2 ***units up.***

Step 2: ***Reflect across the*** x***-axis***

Step 3: ***Rotate counter-clockwise*** ◦90◦ ***about point*** (3,−2)***.***

Help Felicia find out where the lock is by following her steps. The following questions are designed to help you.

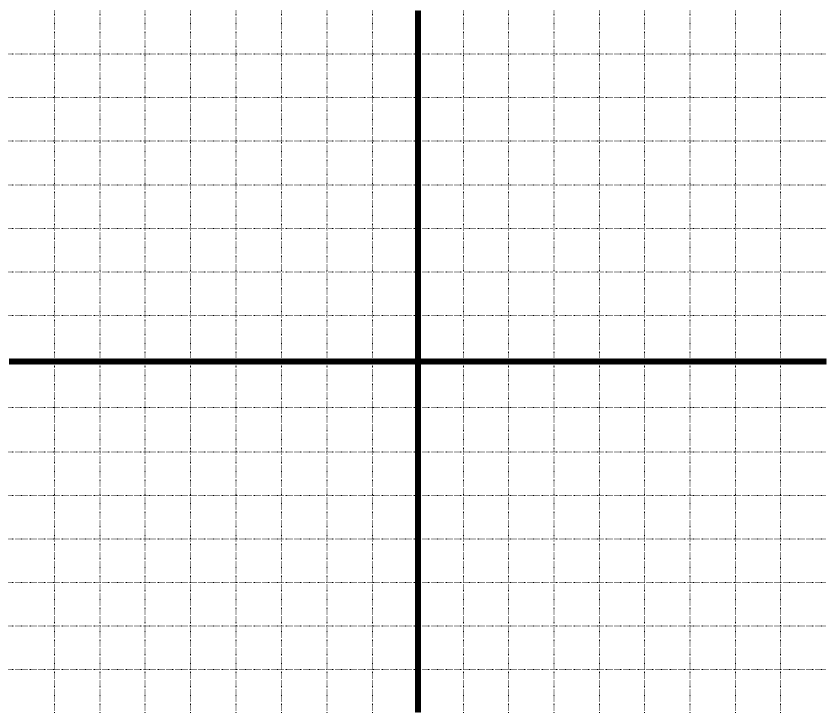
a. With your team, set up your own coordinate grid on graph paper. The questions below will help.
   - How many quadrants (regions) should the graph have? Should it be a graph with only the first quadrant? Or a graph with four quadrants?
   - How should the axes be scaled? How many units should you use for each side length of a grid square?

b. Plot triangle ABC to represent the key.

c. Follow Step 1 to translate the triangle. Name the new location of each **vertex**, or corner, of the triangle in the form (x,y).

d. Complete Step 2. Sketch the triangle in its new position and label the coordinates of each vertex.

e. Where does Felicia's triangle end up? Complete Step 3 on the graph and label the coordinates of each vertex.



Problem 6-10

Now compare the triangle in problem 6-9 that you have after Step 3 with the original triangle. How do the lengths of the sides compare? How do the sizes of the angles compare?

Problem 6-11

Could Felicia's team have used different steps to "unlock" her puzzle in problem 6-9? In other words, could she have used different moves and still have the key end up in the same final position?

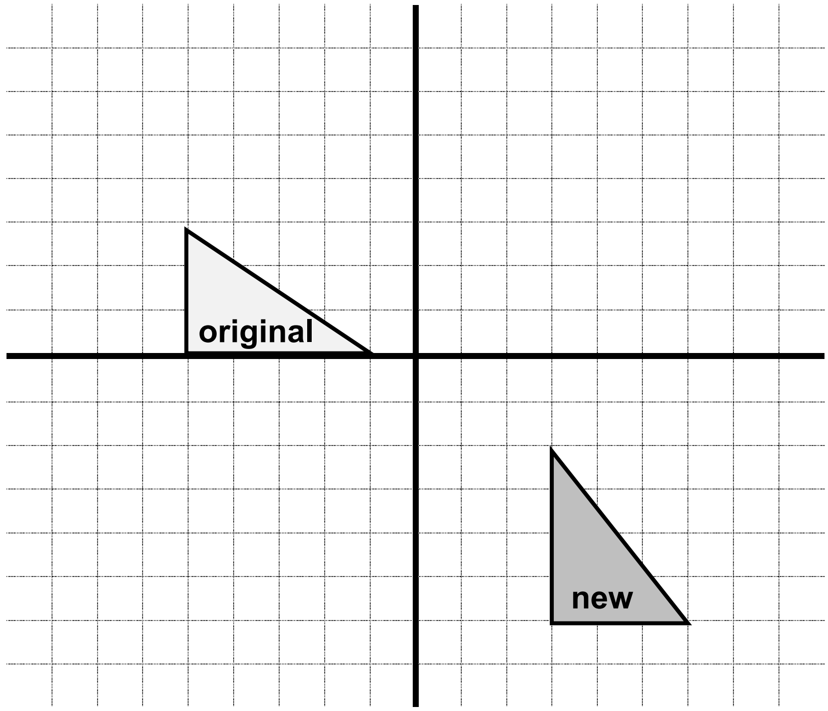
- List a new set of steps that would move her key from the same starting location to the same final position.

1.

2.

3.

4.



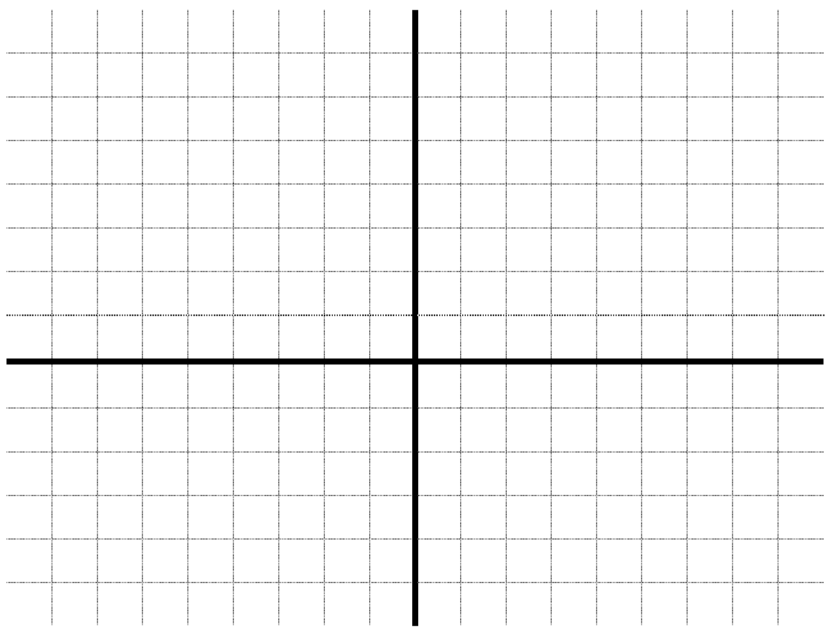
Review & Preview — Problem 6-12

On graph paper, draw a coordinate graph with x- and y-axes. Graph shapes A, B, and C as described below.

a. Shape A is a triangle with vertices (1,1), (3,3), and (2,4).

b. Shape B is a square with vertices (2,−1), (4,−1),(2,−3), and (4,−3).

c. Shape C is a rectangle with vertices (−3,1), (−3,4), (−1,4), and (−1,1).



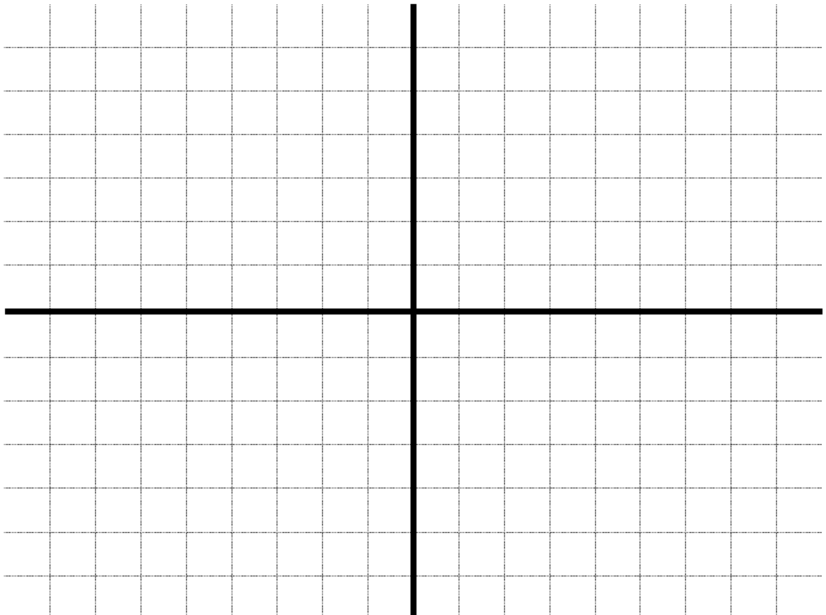
Problem 6-13

On the same grid you used in problem 6-12, translate triangle A four units right and three units up to create triangle D. Write the coordinates of the new vertices.

Problem 6-14

Graph each equation below on the same set of axes and label the point of intersection with its coordinates. Remember to use what you know about the ***m*** and ***b*** from ***y=mx+b***

$$y = 2x + 3 \qquad\qquad y = x + 1$$



Problem 6-15

Shooter Marilyn is the Spartans' best free-throw shooter. She normally makes three out of every four shots. In an upcoming charity event, Shooter will shoot 600 free-throws. If she makes over 400 baskets, the school wins $1000. Should the Spartans expect to win the cash for the school? Show and organize your work.

Problem 6-16

Examine the tile pattern shown below

a. Draw Figure 0 and Figure 4.
b. How many tiles will Figure 10 have? Justify your answer.

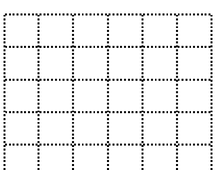


Figure 0

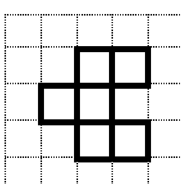


Figure 1

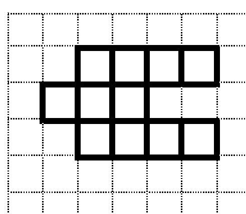


Figure 2

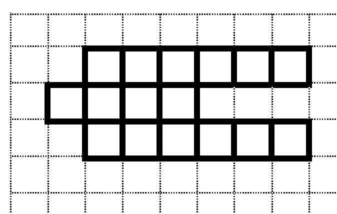


Figure 3

Figure 10

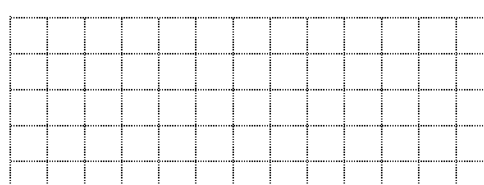


Figure 4

Problem 6-17

GETTING IN SHAPE
Frank weighs 160 pounds and is on a weight plan to gain two pounds a week so that he can make the football team. John weighs 208 pounds and is on a weight plan to lose three pounds a week so that he can be on the wrestling team in a lower weight class.

a. If Frank and John can meet these goals with their weight plans, when will they weigh the same, and how much will they weigh at that time?
b. Clearly explain your method.

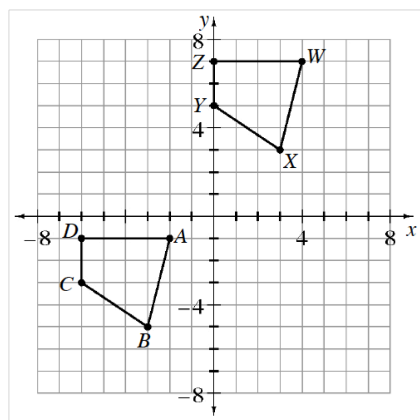
Lesson 6.1.3
Describing Transformations

How can I describe it?

In Lesson 6.1.2, you used words and coordinate points to describe how a triangle moved on a graph. These expressions described the starting place, the motion, and the point where the triangle ended up. Today, you will write similar expressions to describe transformations on a grid.

Problem 6-18

Rosa changed the position of quadrilateral ABCD to that of quadrilateral WXYZ. "*How did the coordinates of the points change?*" she wondered.



a. Describe how Rosa transformed ABCD. Was the shape translated (slid), rotated (turned), or reflected (flipped)? Explain how you know.

b. How far did ABCD move? In which direction?

c. Point *B* became point X. What are the coordinates of points B and X? Name them using (x,y) notation.

d. How did the x-coordinate of point B change? How did its y-coordinate change? For each coordinate, write an equation using addition to show the change.

e. Visualize translating WXYZ 10 units to the right and 12 units up. Where will point X end up? *Without counting on the graph*, work with your team to find the new coordinates of point X. Write equations using addition to show the change.

Problem 6-19

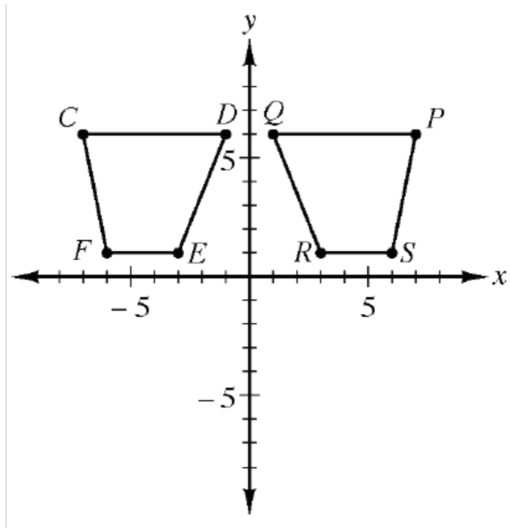
Rosa translated a different shape on a grid. Use the clues below to figure out how her shape was moved.

a. The point (4,7) was translated to (32,−2).
Without graphing, describe how the shape moved on the grid.

b. Another point on her original shape was (−16,9). After the same translation, where did this point end up? For each coordinate, write an expression using addition to show the change.

Problem 6-20

Rowan transformed quadrilateral CDEF below to get the quadrilateral PQRS.



a. Describe how Rowan transformed the quadrilateral. Was the shape translated, rotated, or reflected? Explain how you know.

b. Rowan noticed that the y-coordinates of the points did not change. What happened to the x-coordinates? Compare the x-coordinate of point C with the x-coordinate of point P. What do you notice?

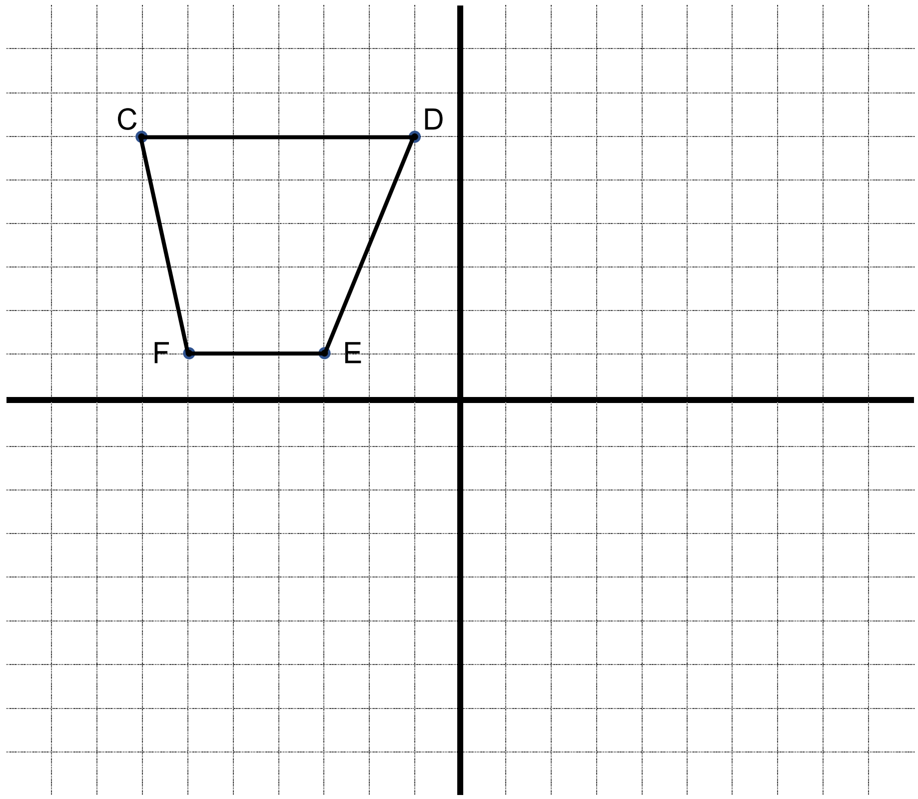
c. Can you describe the change to all of the x-coordinates with addition like you did in problems 6-18 and 6-19? If not, what other operation could you use? Explain.

Problem 6-21

Imagine that Rowan reflected quadrilateral CDEF from problem 6-20 across the x-axis instead. What do you think would happen to the coordinates in that case?

a. First visualize how the quadrilateral will reflect across the x-axis.

b. Reflect quadrilateral CDEF across the x-axis to get quadrilateral JKLM.

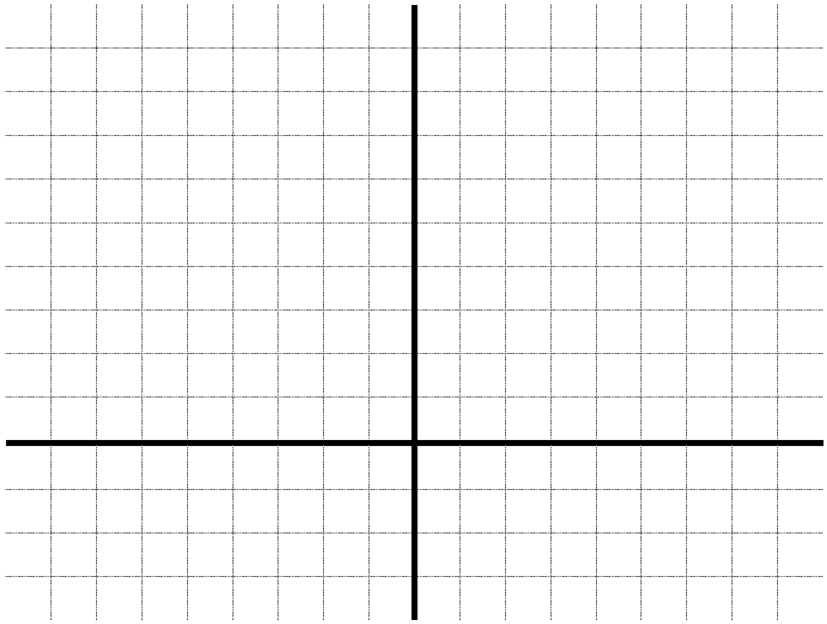


c. Compare the coordinates of point *C* with point J, point D with point K, point E with point L, and point F with point M. What do you notice? How can you use multiplication to describe this change?

Problem 6-22

In problem 6-20, Rowan noticed that multiplying the x-coordinates by −1 reflects the shape across the y-axis.

a. Test this strategy on a triangle formed by the points A (−3,5), B (1,2), and C (0,8). Before you graph, multiply each x-coordinate by −1. What are the new points?
b. Graph your original and new triangle on a new set of axes. Did your triangle get reflected across the y-axis?



Problem 6-23

In the last three lessons, you have investigated rigid transformations: reflections, rotations, and translations. What happens to a shape when you perform a rigid transformation?

Do the side lengths or angles in the figure change?

Do the relationships between the lines (parallel or perpendicular) change?

Why do you think reflections, rotations, and translations are called rigid transformations?

Methods and Meanings
Math Notes

"Rigid Transformation"

Tool Kit page 42

Rigid transformations are ways to move an object while not changing its shape or size. Specifically, they are translations (slides), reflections (flips), and rotations (turns). Each movement is described at right.

A **translation** slides an object horizontally (side-to-side), vertically (up or down), or both. To translate an object, you must describe which direction you will move it, and how far it will slide. In the example at right, Triangle A is translated 4 units to the right and 3 units up.

A **reflection** flips an object across a line (called a **line of reflection**). To reflect an object, you must choose a line of reflection. In the example at right, Triangle A is reflected across the □-axis.

A **rotation** turns an object about a point. To rotate an object, you must choose a point, direction, and angle of rotation. In the example at right, Triangle A is rotated 90° clockwise (↻) about the origin (0,0).

Review & Preview **Problem 6-27**

Erin started with one corner of a figure located at $(-4,5)$ and translated it to end at $(6,8)$. To find out how far the shape moved horizontally, she decided to find the difference between the two x-coordinates. **She wrote: 6−(−4).**

a. When Erin simplified $6-(-4)$ she got 2 as her answer. Is this correct? If not, what is the correct simplification?

b. Write another expression to find out how far the shape moved vertically (↕). Simplify both expressions and describe the translation.

c. Describe each of the translations below.

i. $(3,-2)\rightarrow(5,-9)$

ii. $(-1,4)\rightarrow(6,-2)$

iii. $(0,0)\rightarrow(-4,-7)$

iv. $(-2,-9)\rightarrow(2,9)$

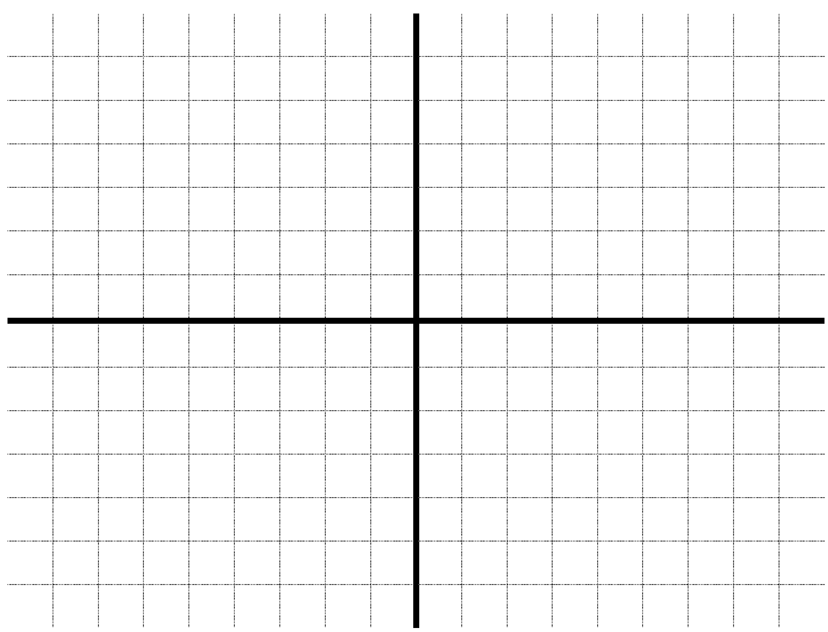
Problem 6-28

Draw a triangle with vertices at $(1,1),(5,1)$, and $(6,3)$. Label this triangle T.

a. Translate (slide) the triangle left 3 units and down 4 units. Label this triangle A and list the vertices.

b. Reflect triangle T across the y-axis. Label this triangle B and list the vertices.

c. Are triangles T, A, and B **congruent** (that is, do they have the same shape and size)? Explain.



Problem 6-29	
Change each equation below into y=mx+b for	
a. $y - 4x = -3$	b. $3y - 3x = 9$
c. $3x + 2y = 12$	d. $2(x - 3) + 3y = 0$

Problem 6-30
Solve the problem below by defining a variable and then writing and solving an equation. The perimeter of a triangle is 31 cm. Sides #1 and #2 have equal length, while Side #3 is one centimeter shorter than twice the length of Side #1. How long is each side?

Problem 6-31

Simplify each expression.

a. $$\frac{73}{100} \cdot -\frac{2}{7}$$

b. $$0.4 \cdot 0.3$$

c. $$5\frac{1}{9} + 8\frac{2}{5}$$

d. $$-1.2 + (-\frac{3}{5})$$

Lesson 6.1.4
Using Rigid Transformations

What Can I Create?

In the last several lessons, you have described translations using coordinates. You have also developed strategies for determining where an object started when you know how it was translated and its final position. In this lesson, you will continue to practice transforming objects on a coordinate graph by translating (sliding), rotating (turning), and reflecting (flipping). As you work, **visualize** what each object will look like after the transformation. Use the graph to check your prediction.

Problem 6-33

BECOMING AN ARTIST

Your Task:

-On the next page, shapes A, B, and C, are provided for you.

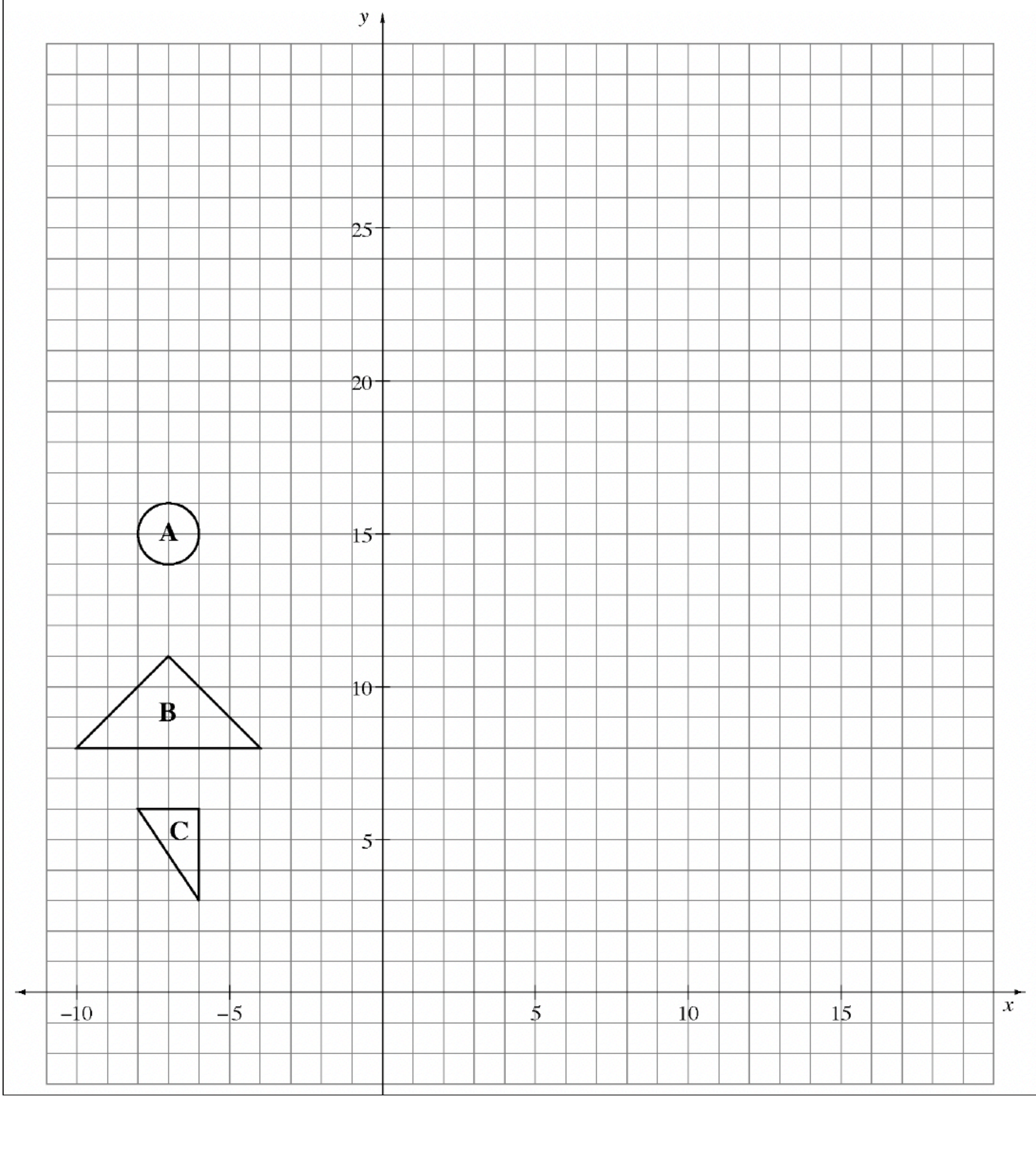
-Follow the directions at the top of the page to create a picture.

-Whenever one of the shapes is mentioned below, start with the original shape on the left side of the paper.

-The final result of each step is part of the outline of the picture.

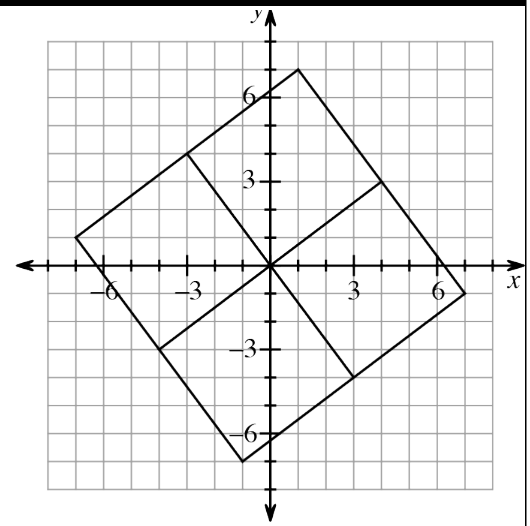
Once you have finished following all the directions, complete the picture with color and other details.

1. Draw a rectangle with vertices (5,0), (9,0), (9,8), and (5,8).
2. Translate circle A so that its center is at (7,6).
3. Rotate triangle C 180° clockwise around the point (−6,3). Record the coordinates of the new vertices. Then add 15 to each x-coordinate and graph the final result. What transformation does “adding 15” represent?
4. Reflect (flip) triangle B across the y-axis.
5. A new shape, triangle D, has vertices at (−7,13),(−8,11), and (−6,11). Translate triangle D so that its top vertex is at (7,4). Describe this translation with words.
6. Translate triangle C to the right 11 units. Then reflect the result across the horizontal (⇔) line that goes through y=3. Record the new coordinates for triangle C.
7. Translate circle A so that the x-coordinates increase by 13 units and the y-coordinates increase by 11 units. Record the coordinates of the center of circle A in its new position.



Problem 6-35

Examine the figure on a coordinate graph as shown at right.
What transformations could you apply to the entire figure so that the sides of the squares line up exactly with the grid lines of the coordinate graph?



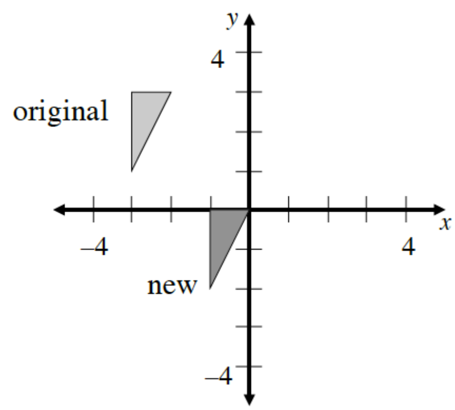
Review & Preview — Problem 5-36

Use the graph at right.

a. Write directions to translate the original triangle to make the new triangle.

b. What are the coordinates of the vertices (corners) of the new shape?

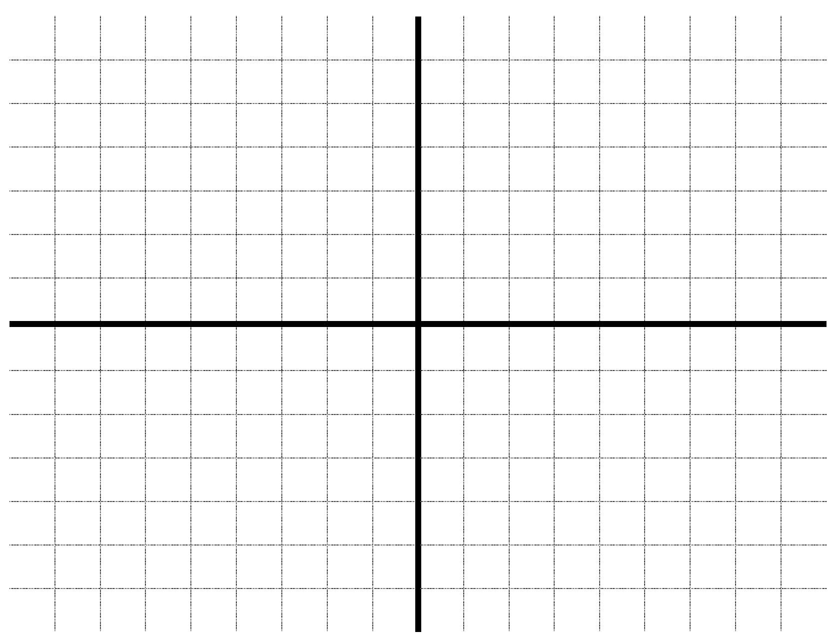
c. On your graph, reflect the original triangle across the y-axis. What are the coordinates of the vertices of the new triangle?



Problem 6-37

Make a table and graph for the rule $y = -3x + 1$.

x	$y = -3x + 1$



Problem 6-38

Solve the **system of equations** below using the Equal Values Method.

$$a = 12b + 3 \qquad\qquad a = -2b - 4$$

Problem 6-39

Ms. Cai's class is studying a tile pattern. The rule for the tile pattern is $\boldsymbol{y = 10x + 8}$. Kalil thinks that Figure 12 of this pattern will have 108 tiles. Is he correct? Use the equation to justify your answer.

Problem 6-40

Angel is picking mountain blueberries for a delicious pie. She can pick $\frac{1}{6}$ cup of blueberries in 2 minutes. If she needs $\mathbf{2}\frac{\mathbf{1}}{\mathbf{2}}$ cups of blueberries for the pie, how long will it take her to pick the berries?

Problem 6-41

Juan thinks that the graph of $6y + 12x = 4$ is a line.

a. Solve Juan's equation for y.

b. Is this equation linear? That is, is its graph a line? Explain how you know.

c. What is the pattern of growth and y-intercept of this graph?

Lesson 6.2.1
Multiplication and Dilation

What if I Multiply?

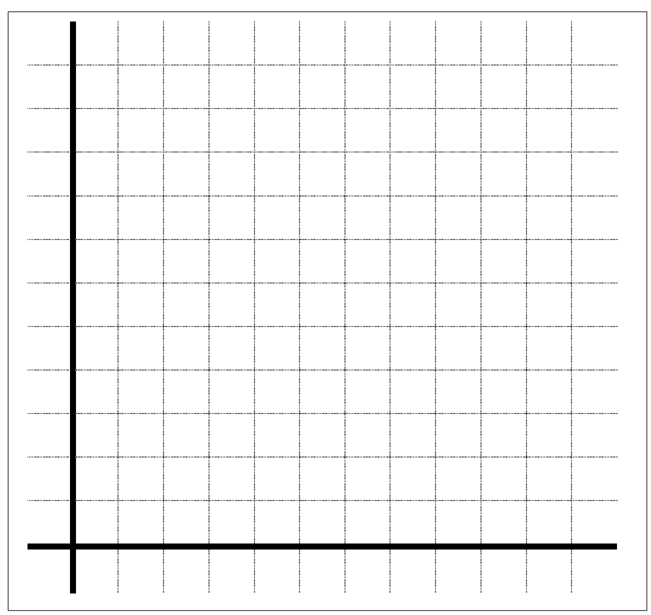
Remember that when an object is translated, rotated, or reflected, it stays the same size and shape even though it moves. For this reason, these types of movements are called rigid transformations. In this lesson, you will explore a new transformation that changes how the object looks. As you work today, ask these questions in your team:

What parts of the shape are changing? What parts stay the same?

Problem 6-42

When all of the x- or y-coordinates of each vertex of a shape are changed by adding or subtracting the same numbers, the shape translates (slides) to a new position on the coordinate graph. You learned in Lesson 6.1.3 that when one of the coordinates of the vertices (either the x- or y-coordinate) is multiplied by −1, the shape is reflected over the y-axis or the x-axis, respectively.

How do you think the shape will change when both the x- and y-coordinates are multiplied by some number? Use the directions below to help you answer this question.



a. Plot the following points on graph paper: (2,1),(3,1),(5,5),(2,5). Connect the points to make a quadrilateral.

b. Without graphing, predict how you think the figure would change if the x- and y-coordinates were multiplied by 2 and then plotted. (Discuss with Team)
( ) ( ) ( ) ( )

c. Test your prediction by doubling the coordinates from part (a) and plotting them on your graph paper. Was your prediction correct?

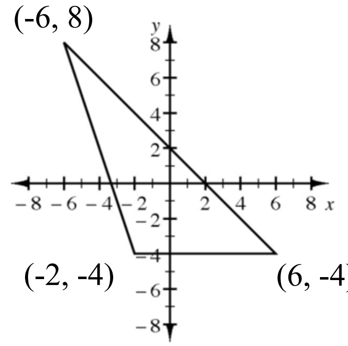
With your team members, look at the figure you just graphed. Transforming a graphed shape by multiplying each coordinate by the same number is called a **dilation**. With your team, discuss how this figure compares to the original. Be specific about changes in, side length and area!

How did the side length change? *How did the area change?*

Problem 6-43

INVESTIGATING DILATIONS

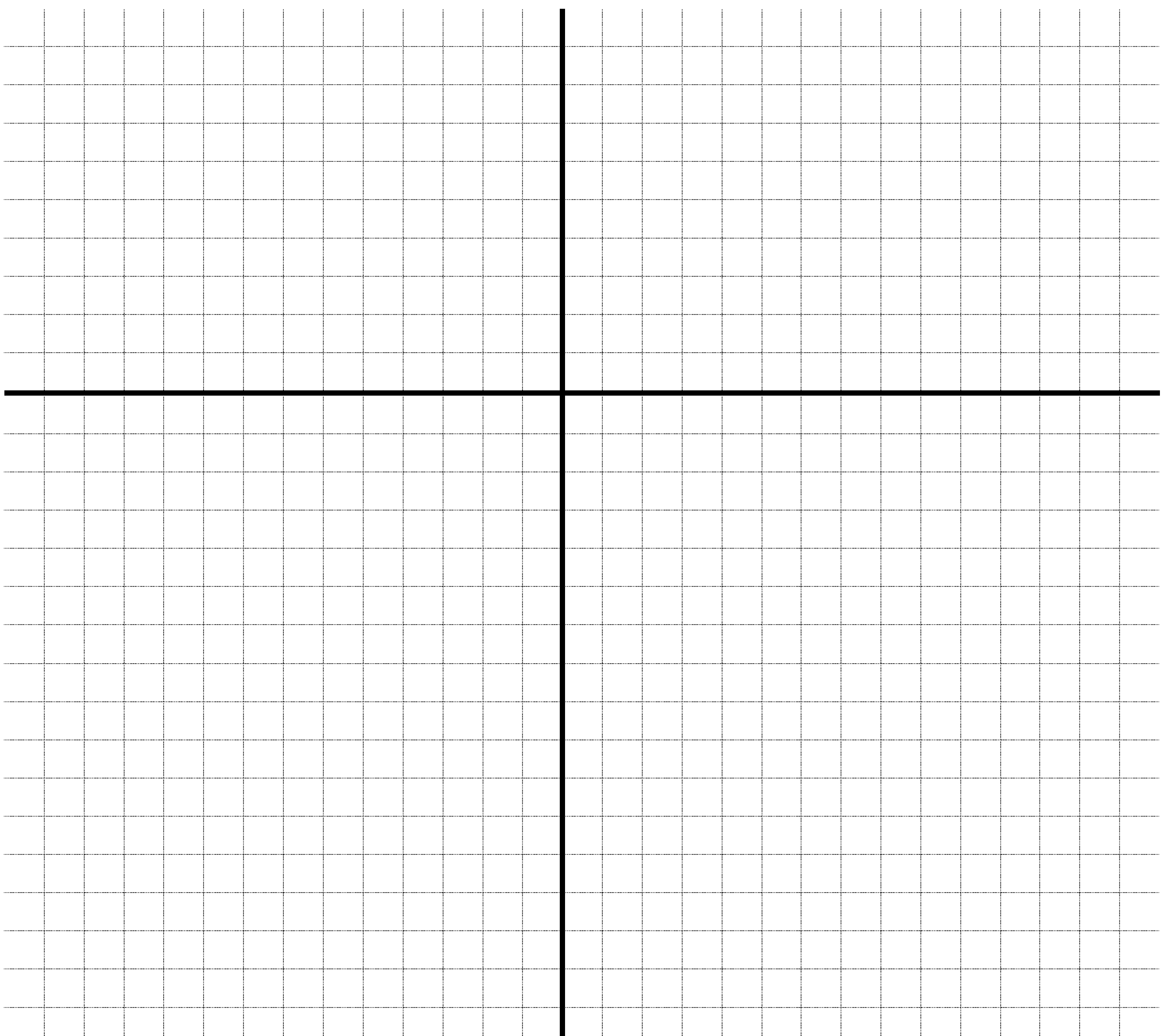
The students in Ms. Stanley's class were studying what happens to the graph of a shape when both coordinates are multiplied by the same number. They came up with these questions:



- "*What happens when each coordinate is multiplied by one half*?"
  Write the new coordinates. (   ) (   ) (   )
- *"What happens when both of the coordinates are multiplied by* −1?"
  Write the new coordinates. (   ) (   ) (   )
- *"What happens when the coordinates are multiplied by* −2*?"*
  Write the new coordinates. (   ) (   ) (   )

Use the shape at right to investigate the questions above. Use graph paper to make the dilations.

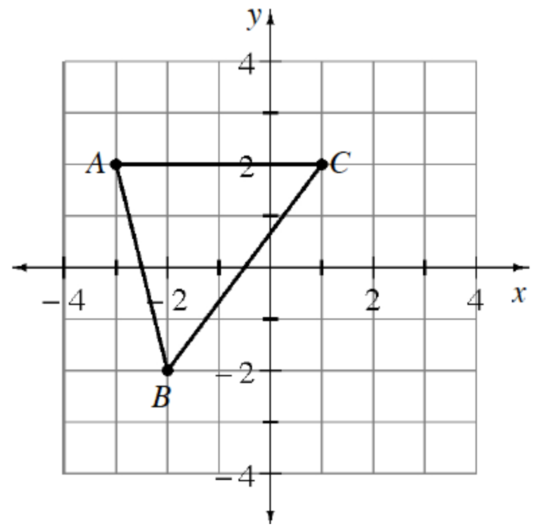
How did the figure change in each of the investigations? Compare the side lengths, the angles, and the line relationships. Explain what your team learned about the three questions you investigated.

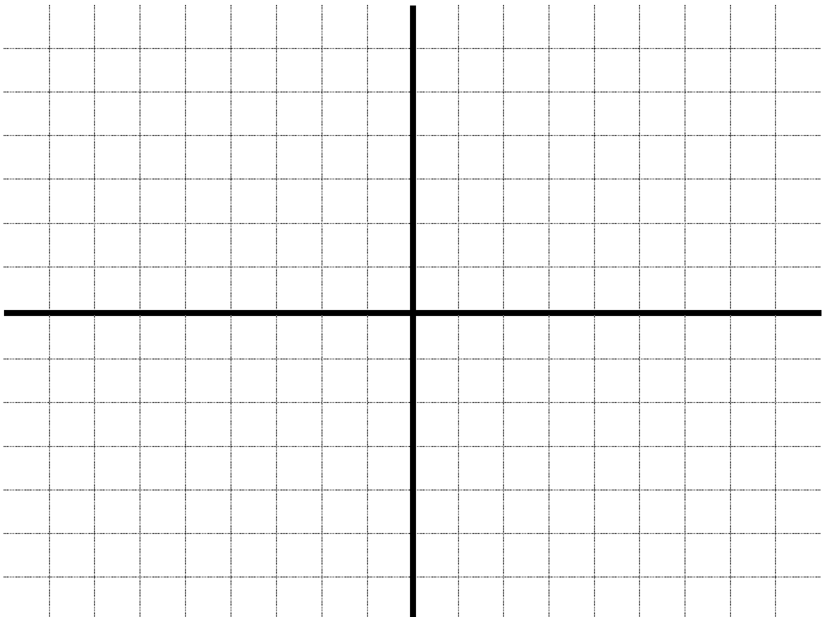


Review & Preview **Problem 5-46**

Louis is dilating triangle ABC at right. He multiplied each x-coordinate and y-coordinate of triangle ABC by −2.

a. What are the new coordinates of the points?
b. Graph Louis' new triangle.
c. Describe how triangle ABC changed.

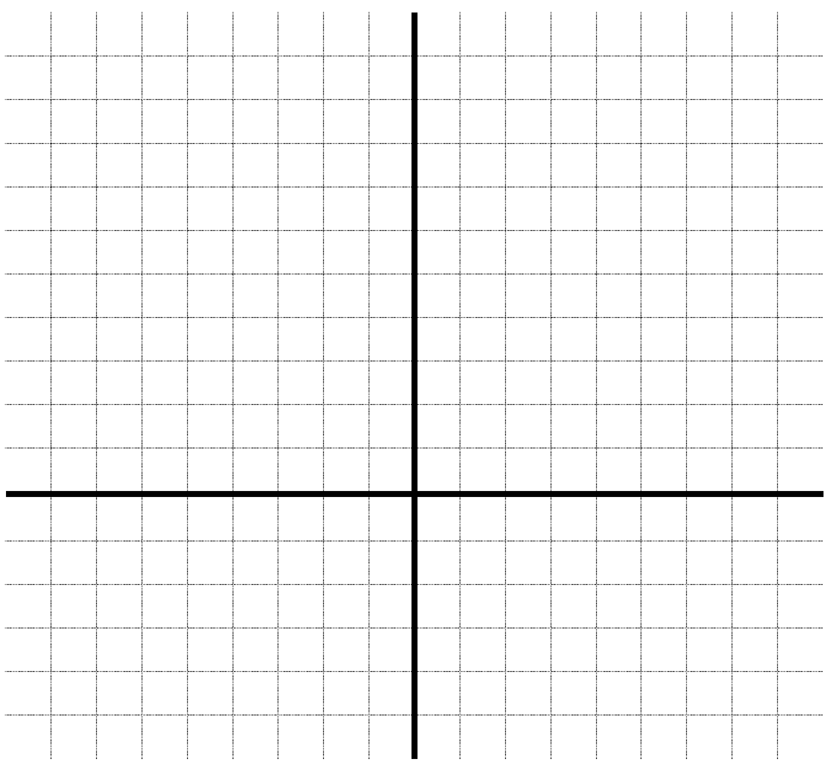




Problem 6-47

On the same set of axes, graph the two rules shown at right. Then find the point(s) of intersection, if one (or more) exists.

$$y = -x + 2 \qquad y = 3x + 6$$



Problem 6-48

Evaluate the expression $\mathbf{6x^2 - 3x + 1}\ for\ \mathbf{x = -2}$.

Problem 6-49

When Ms. Shreve solved an equation in class, she checked her solution, and it did not make the equation true!

Examine her work below and find her mistake. Then find the correct solution.

$$\mathbf{5(2x-1) - 3x = 5x + 9}$$

$$\mathbf{10x - 5 - 3x = 5x + 9}$$

$$\mathbf{7x - 5 = 5x + 9}$$

$$\mathbf{12x = 4}$$

$$\mathbf{x = \frac{1}{3}}$$

Problem 6-50

Determine if the statement below is true or false.   Explain your conclusion.

$$\mathbf{2(3 + 5x) \Rightarrow 6 + 5x}$$

Problem 6-51

Complete the missing entries in the table below. Then write the rule.

IN *(x)*	2	10	6	7	-3		-10	100
OUT *(y)*	4	28	16			10		

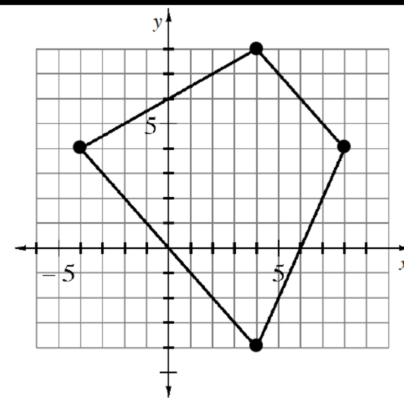
y=

Lesson 6.2.2
Dilations and Similar Figures *How do shapes change?*

Have you ever wondered how different mirrors work? Most mirrors show you a reflection that looks just like you. But other mirrors, like the mirrors commonly found at carnivals and amusement parks, reflect back a face that is stretched or squished. You may look taller, shorter, wider, or narrower. These effects can be created on the computer if you put a picture into a photo program. If you do not follow the mathematical principles of proportionality when you enlarge or shrink a photo, you may find that the picture is stretched thin or spread out, and not at all like the original. Today you will look at enlarging and reducing shapes using dilations to explore why a shape changes in certain ways.

Problem 6-52

UNDOING DILATION
In Lesson 6.2.1, you looked at dilations and multiplied each of the coordinates of a shape to change its size. Now you will explore how to undo dilations.



Charlie multiplied each coordinate of the vertices of a shape by 4 to create the dilated shape at right.

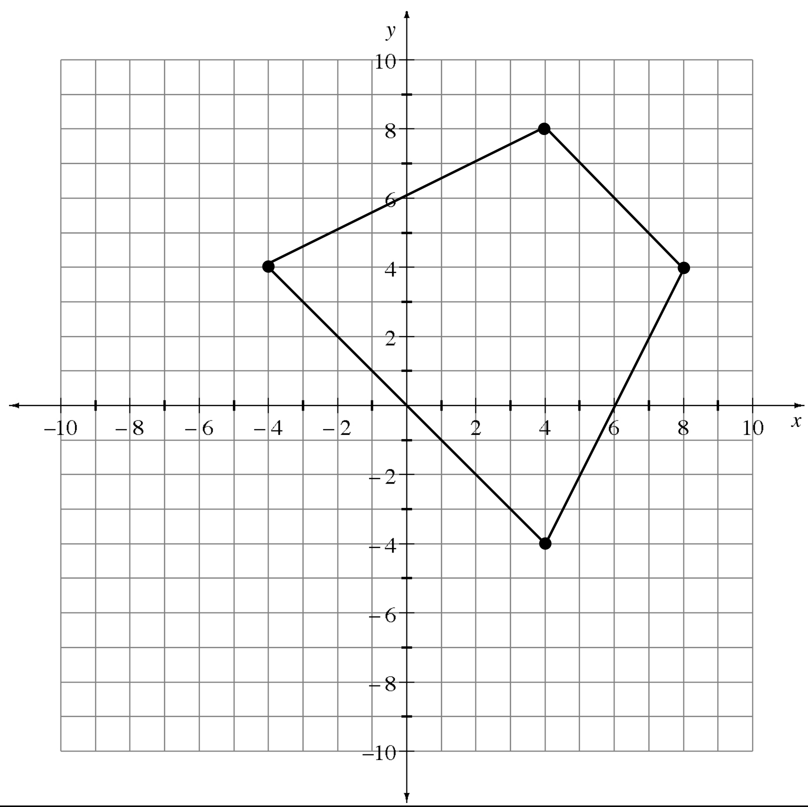
a. If Charlie multiplied to find this shape, what operation would undo his dilation? Why?

b. On <u>the below grid</u>, undo the dilation on the graph at right. Label the vertices of Charlie's original shape.

How does the shape compare to the dilated shape?

How did the dimensions change?

How did the area change:

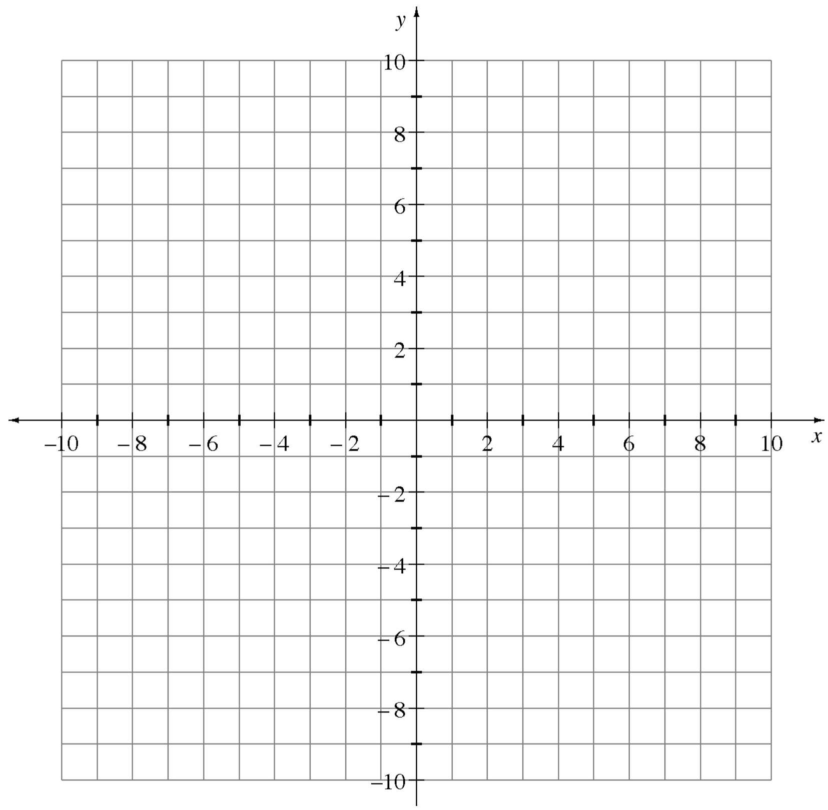


Problem 6-53

Alana was also working with dilations. She wondered, *"What would happen if I multiplied each coordinate of a shape by $\frac{1}{3}$?"* On the grid below, graph and connect the below points to form her dilated shape. Be sure to connect them in the order given. (−1,−1) (−1,1) (1,2) (2,−1)

a. Alana graphed this shape by multiplying each of her original coordinates by $\frac{1}{3}$. What do you think Alana's shape looked like before the dilation? Make a prediction.

b. On the same graph, undo the dilation to show Alana's original shape. List the coordinates of the vertices of Alana's original shape.



c. What did you do to each coordinate to undo the dilation? How did the shape change?

d. Why do you think the shape changed in this way?

Problem 6-54

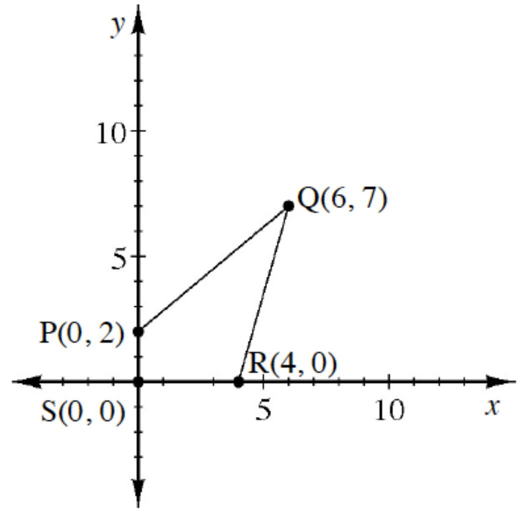
With your team, look carefully at Alana's dilated and original shapes in problem 6-53 and describe how the two shapes are related. Use the questions below to help you.

- How are the sides of the small and large shape related?
- How many of the small sides does it take to measure the **corresponding** (matching) side of the large shape? Is this true for all of the sides?
- Compare the four angles of the smaller shape to those of the larger shape. What can you say for sure about one matching pair of these angles? What appears to be true about the other three pairs?

Problem 6-55

CHANGING SHAPE

When you multiplied each coordinate of a shape by the same constant, you saw that sometimes the shape became smaller and sometimes it became larger. In this chapter, you moved shapes on a graph without changing their size or shape by rotating, reflecting, and translating them. In what other ways can you change a shape?



Your Task: Work with your team to make predictions about what you could do to the coordinates of the shape at right to make it look stretched or squished, and what actions will keep the shape the same. Make predictions for the situations presented below. You will test these predictions in problem 6-56.

<u>Discussion Points</u>

What do you think will change if both the x-and y-coordinates of the points P, Q, R, and S are multiplied by the same number, such as 4?

What do you think will happen if only the x-coordinates are multiplied by 3?

What do you think will happen if just the y-coordinates are multiplied by 2?

What do you think will happen if the x-and y-coordinates are multiplied by different numbers, like 2 for x and 3 for y?

Problem 6-56

Test the predictions your team made in problem 6-55. Using the graph below, graph each of the shapes described below.

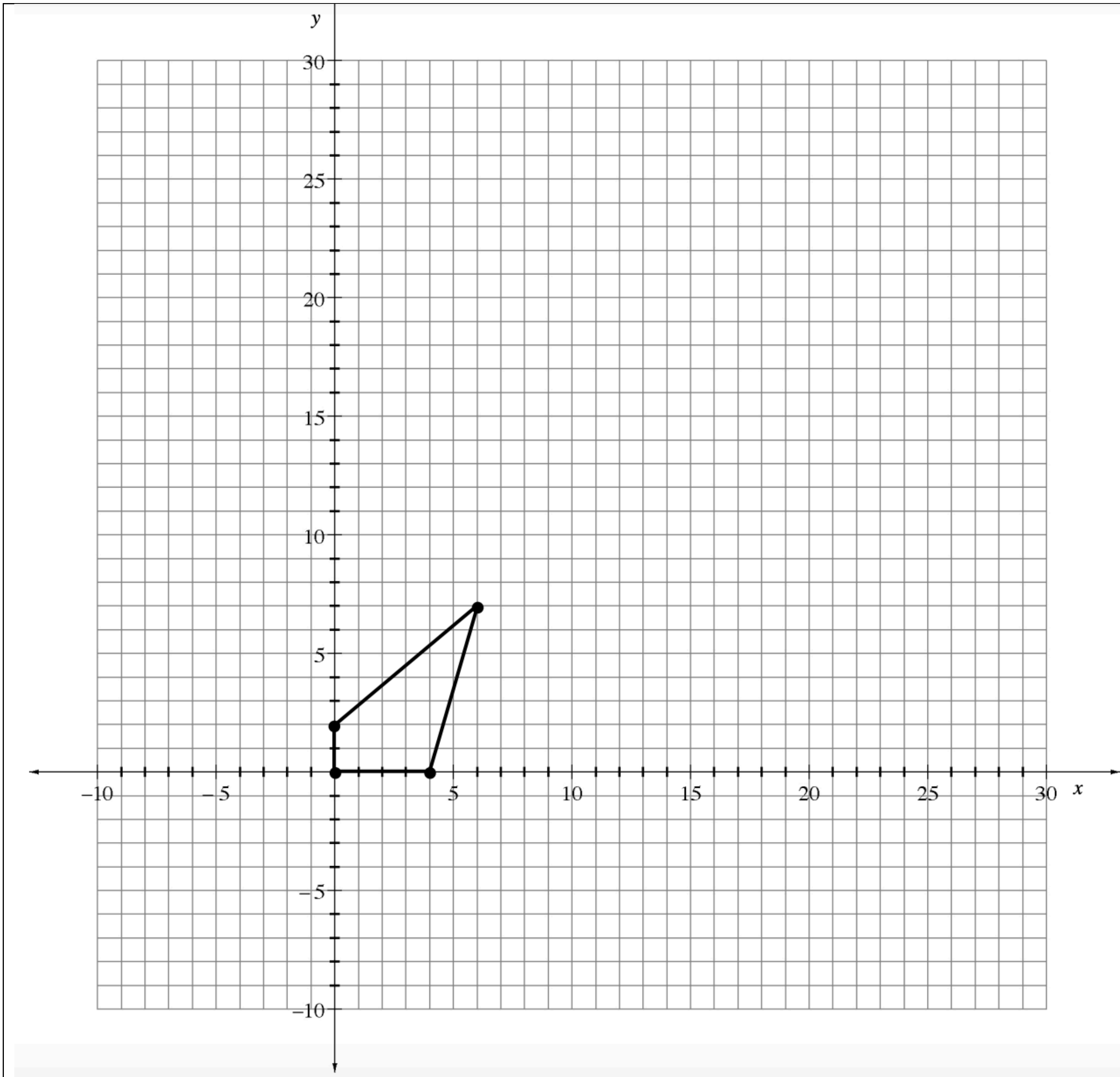
a. **Dilate each coordinate of shape *PQRS* by multiplying each x-coordinate and each y-coordinate by 4.** Graph the dilated shape on the same graph using a color other than black.

b. Go back to the original shape, and this time **multiply only the x-coordinates by 3**. Leave the y-coordinates the same. Find, graph, and connect the new coordinates.

c. What happened to the shape in part (b)? Why did this happen?

d. Look at the predictions your team made in problem 6-55. Do you still agree with your predictions? Revise your predictions, if necessary, based on your work so far. What do you think will happen if you multiply only the y-coordinates of the vertices by a number? Be ready to explain your reasoning.

Problem 6-56 continued



Problem 6-57

Similar figures are figures that have exactly the same shape, but not necessarily the same size. One characteristic of similar shapes is that the sides of one shape are each the same number of times bigger than the corresponding sides of the smaller shape.

Which pairs of shapes that you have worked with in this lesson are similar and which are not? **Justify** your answer using specific examples.

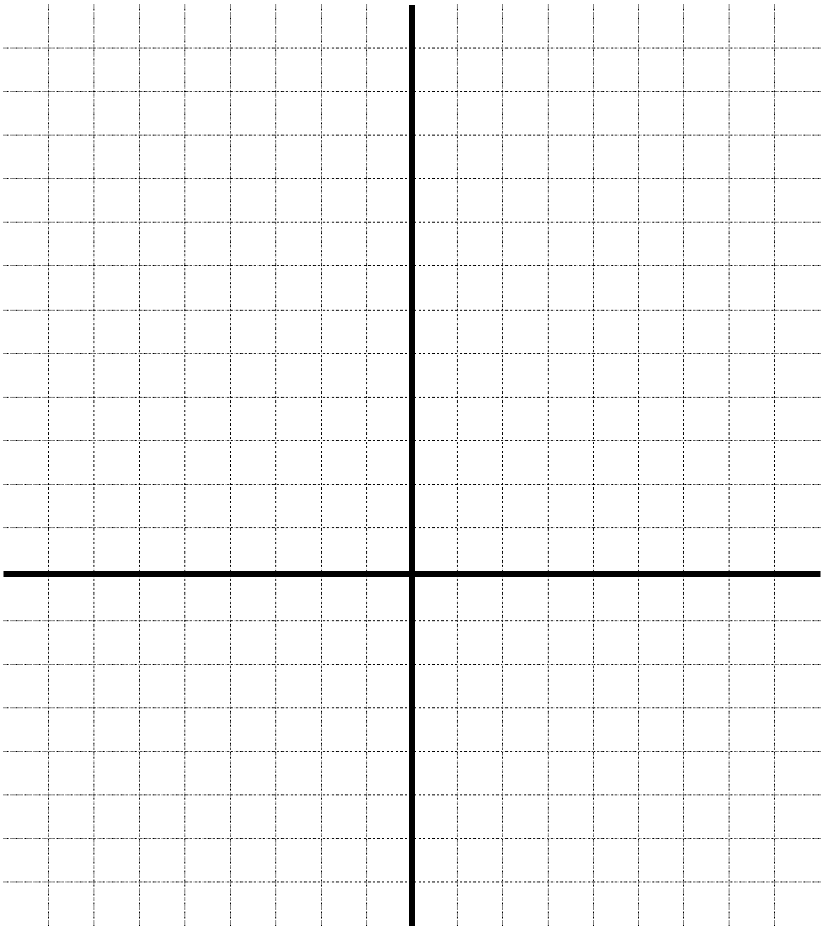
METHODS AND MEANINGS MATH NOTES

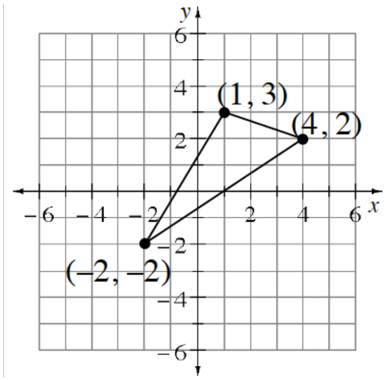
"Corresponding Parts of SIMILAR Shapes"

Tool Kit page 43

Review & Preview — Problem 6-58

Graph the triangle at right. **Multiply the y-coordinate of each point by 4**. Then graph the new shape. Make sure you connect your points. List the points for the new shape.

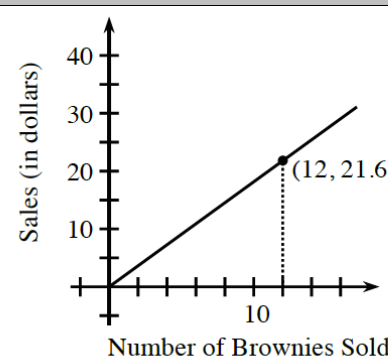




Are the two figures similar? Why or why not?

Problem 6-59

Lashayia is famous for her delicious brownies, which she sells at football games. The graph at right shows the relationship between the number of brownies she sells and the amount of money she earns.



a. How much should she charge for 10 brownies? Be sure to demonstrate your reasoning.

b. During the last football game, Lashayia made $34.20. How many brownies did she sell? Show your work.

Problem 6-60

Figure 3 of a tile pattern has 11 tiles, while Figure 4 has 13 tiles. The pattern grows at a constant rate.

a. Define the variable and write an equation to represent this situation.

b. Use the equations you wrote to determine which figure will contain 1015 tiles?

Problem 6-61

Normally, the longer you work for a company, the higher your salary per hour. Hector surveyed the people at his company and placed his data in the table below.

Number of Years at Company	1	3	6	7
Salary per Hour	\$7.00	\$8.50	\$10.75	\$11.50

a. Use Hector's data to estimate how much he makes, assuming he has worked at the company for 12 years.

b. Hector is hiring a new employee who will work 20 hours a week. How much should the new employee earn for the first week?

Problem 6-62

Mr. Greer solved the equation as shown below. However, when he checked his solution, it did not make the original equation true. Find his error and then find the correct solution.

$$4x = 8(2x - 3)$$

$$4x = 16x - 3$$

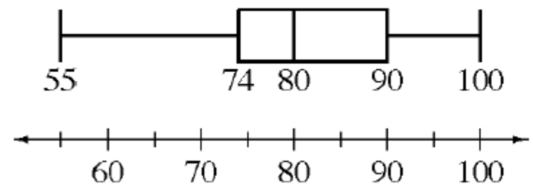
$$-12x = -3$$

$$x = \frac{-3}{-12}$$

$$x = \frac{1}{4}$$

Problem 6-63

The box plot at right shows the different grades (in percents) that students in Ms. Sanchez's class earned on a recent test.



a. What was the median score on the test? What were the highest and lowest scores?

b. Did most students earn a particular score? How do you know?

c. If Ms. Sanchez has 32 students in her class, about how many students earned a grade of 80% or higher? About how many earned more than 90%? Explain how you know.

d. Can you tell if the scores between 80% and 90% were closer to 80% or closer to 90%? Explain.

Lesson 6.2.3
Identifying Similar Shapes

Are they similar?

Have you ever noticed how many different kinds of cell phones there are? Sometimes you might have a cell phone that is similar to one of your friends' cell phones because it is the same brand, but it might be a different model or color.

Occasionally, two people will have the exact same cell phone, including brand, model, and color. Sorting objects into groups based on what is the same about them is also done in math. As you work with your team to sort shapes, ask the following questions:

How do the shapes grow or shrink? What parts can we compare? How can we write the comparison?

Problem 6-64

WHICH SHAPES ARE SIMILAR?

If two shapes appear to have the same general relationship between sides, how can you decide for sure if the shapes are similar? Work with your team to:

- Carefully cut out the original shape and shapes A through G from one copy of the resource page
- Decide how each shape is related to the original shape. (Each person should use an uncut copy of the resource page and the team's cut out shapes to help decide.)
- Compare the angles and the sides of shapes A through G to the original shape.

Problem 6-64 Continued

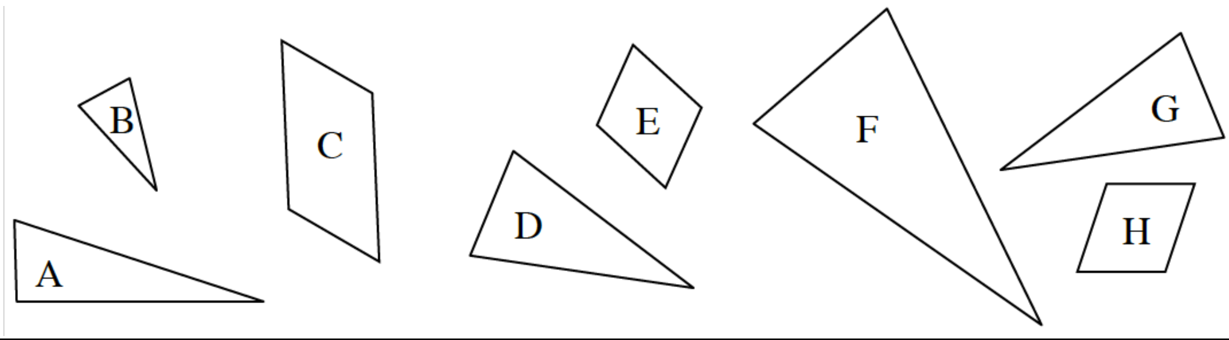
Glue the original shape here.

Similar	NOT Similar
	a. Which shapes are similar to the original shape? Give specific reasons to justify your conclusions. What do these shapes have in common? What is different about them? Be specific b. When two shapes are similar, the scale factor is the number you multiply the length of the side of one shape by to get the length of the corresponding side of the new shape. What is the scale factor between the original shape and all the other similar shapes. Write the scale factor onto each shape. c. What is the scale factor between the original shape and shape C? Why is it less than 1?

Problem 6-65

Shapes that are similar but do not grow or shrink are called congruent shapes.

a. Which shape from problem 6-64 is exactly equal to the original shape in every way?
b. Record the pairs of shapes below that appear to be congruent to each other.
c. Get a piece of tracing paper from your teacher and use it to check that the shapes you identified as congruent have exactly the same size and shape. Were you correct? If not, why not?



Problem 6-66

Quan enlarged shape Q to make shape P, below. Are his shapes similar? If they are similar, identify the scale factor (multiplier). If they are not, demonstrate that at least one pair of sides does not share the scale factor.

Problem 6-67

Color-code the corresponding sides of ***P*** and ***Q*** problem 6-66 using the colors.

a. Compare the corresponding (matching) sides of shapes P and Q. What do you notice about those sides?

b. Imagine enlarging shape P to make a new shape R that has a base 25 units long. If shape R is similar to shape P, predict the length of the shorter vertical side of shape R *without drawing the shape*. What is the scale factor in this situation?

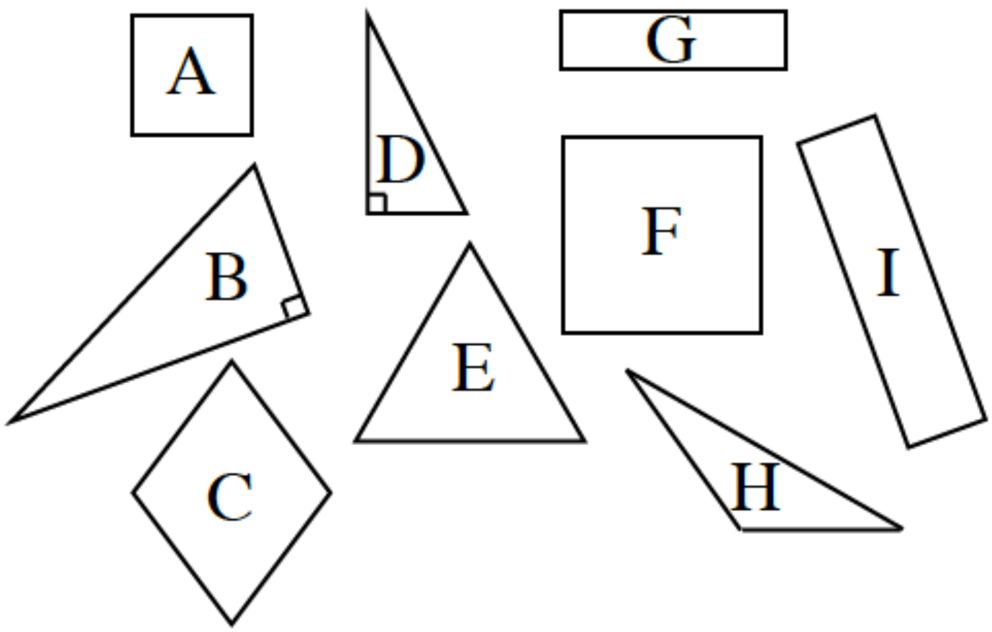
Problem 6-69

LEARNING LOG

In your Learning Log, explain how to determine when shapes are similar. To decide if two shapes are similar, what do you need to know about the side lengths? The angles? Title your entry “Finding Similar Shapes” and include today’s date.

Review & Preview — Problem 6-70

Which of the shapes below appear to be similar? Explain how you know.

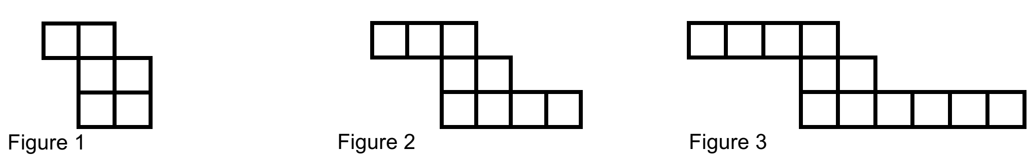


Problem 6-71

A local deli sells 6-inch sub sandwiches for $2.95. Now the deli has decided to sell a “family sub” that is 50 inches long. If they want to make the larger sub price comparable to the price of the smaller sub, how much should it charge? Show all work.

Problem 6-72

Write the rule represented by the tile pattern. (It might help to make a table first.)



Problem 6-73

How many solutions does each equation below have? How can you tell?

a. $4x - 1 + 5 = 4x + 3$

b. $6t - 3 = 3t + 6$

c. $6(2m - 3) - 3m = 2m - 18 + m$

d. $10 + 3y - 2 = 4y - y + 8$

Problem 6-74

Simplify each expression below.

a. $-2\frac{3}{10} - 1\frac{2}{5}$

b. $3 \div \frac{5}{4}$

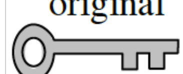
c. $\frac{3}{4} + 5\frac{7}{8}$

d. $5\frac{1}{6} \cdot (-\frac{7}{9})$

Problem 6-75

Look carefully at the key at right.

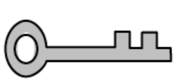
original



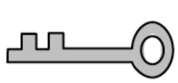
Which of the keys below could you create by turning (rotating) the original key?

Which keys could you create by flipping (reflecting) the original key?

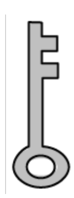
a.



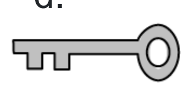
b.



c.



d.



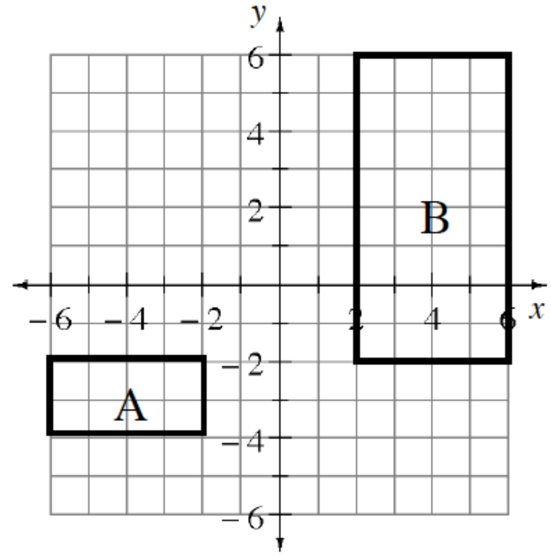
Lesson 6.2.4
Similar Figures and Transformations

What sequence makes them the same?

So far in this chapter you have investigated transformations and similar figures. Recall that reflections, rotations, and translations are all special cases of transformations that are called rigid transformations. Today you will investigate how to use transformations to show that two figures are similar.

Problem 6-76

Examine the graph at right.



a. Do you think the figures are similar? Why or why not?

b. Describe a sequence of transformations (reflections, rotations, translations, and dilations) to change Figure A to Figure B.

c. How does your sequence of transformations prove that the figures are similar?

Problem 6-77

Figures that are **congruent** are the **same shape and the same size**.
You can also say they have a scale factor of 1.

Which transformation(s) can you use to show that two figures are congruent?

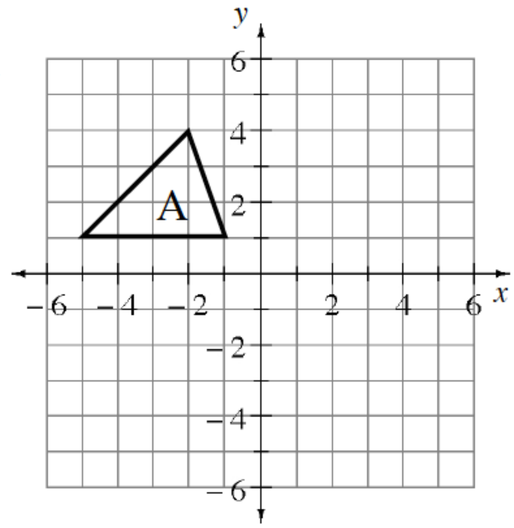
Which transformation(s) will cause figures that are not congruent, but similar?

Problem 6-78

Angelina and Vee have each made a challenge for you. Begin with Figure A at right, and then follow the steps of their transformations to locate the vertices of their new figures. Record your work on the graph.

a. Angelina's steps:

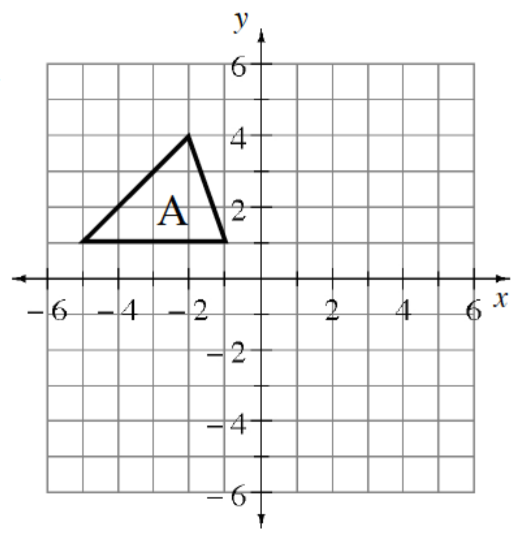
- Reflect the triangle across the x-axis.
- Rotate the triangle about the origin counter-clockwise (↺) º90º.
- Dilate the figure from the origin by a scale factor of $\frac{1}{2}$ (multiply the coordinates of each point by $\frac{1}{2}$).



b. Vee's steps:

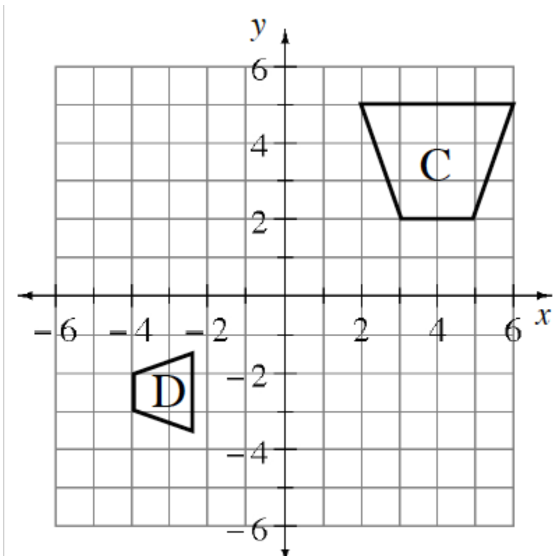
- Translate the triangle 4 units right and 3 units down.
- Rotate the triangle clockwise (↻) º180º about its top vertex.
- Reflect the triangle across the line x=3.

c. Were your resulting figures congruent, similar, or neither? Explain.



Problem 6-79

With your team, find a sequence of transformations that will transform Figure C to become Figure D.



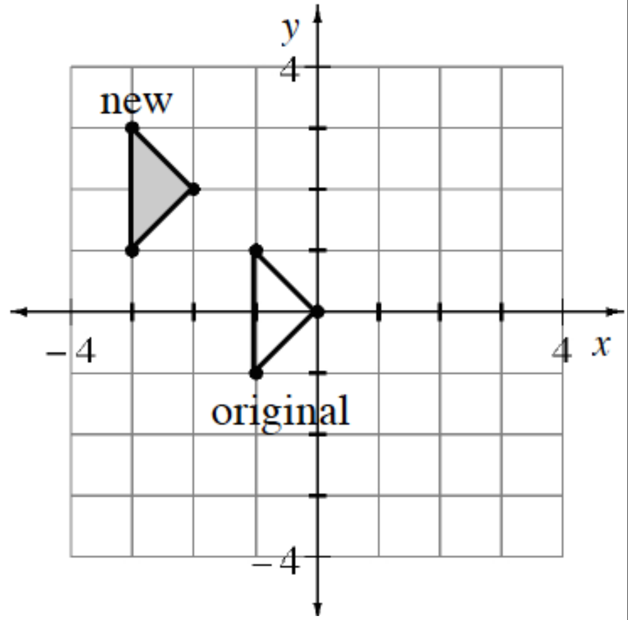
Review & Preview — Problem 6-80

Use graph to the right.

a. Write directions for translating the original triangle to make the new triangle.

b. What are the coordinates of the vertices (corners) of the new shape?

c. On your graph, reflect the original triangle over the y-axis. What are the coordinates of the vertices of the new triangle?



Problem 6-81

Hannah thinks the solution to the system below is *(−4,−6)*. Wirt thinks the solution is *(20,10)*.

$$\begin{aligned} 2x - 3y &= 10 \\ 6y &= 4x - 20 \end{aligned}$$

a. Is Hannah correct?

b. Is Wirt correct?

c. What do the answers to (a) and (b) tell you about the lines in the problem?

Problem 6-82

Figure 2 of a tile pattern is shown at right. If the pattern grows linearly and if Figure 6 has 18 tiles, then find a rule for the pattern.

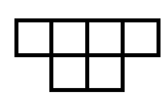


Figure 2

Problem 6-83

Solve the following equations for x, if possible. Check your solutions.

a. $-(2-3x)+x=9-x$

b. $6x+2=34$

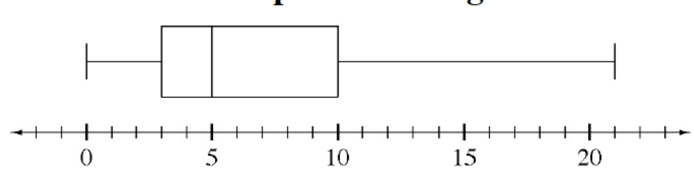
c. $5-2(x+6)=14$

d. $12x-4=-3-13x$

Problem 6-84

Kevin found the box plot below in the school newspaper.

Number of hours spent watching TV each week



a. Based on the plot, what percent of students watch more than 10 hours of television each week?

b. Based on the plot, what percent of students watch less than 5 hours of television each week?

c. Can Kevin use the box plot to find the mean (average) number of hours of television students watch each week? If so, what is it? Explain your reasoning.

Lesson 6.2.5
Working With Corresponding Sides

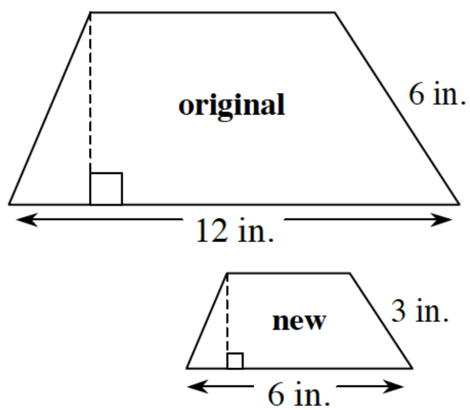
What do Similar shapes tell us?

Graphic artists often need to make a shape larger to use for a sign. Sometimes they need to make a shape smaller to use for a bumper sticker. They have to be sure that the shapes look the same no matter what size they are. How do artists know what the side length of a similar shape should be? That is, does it need to be larger or smaller than the original?

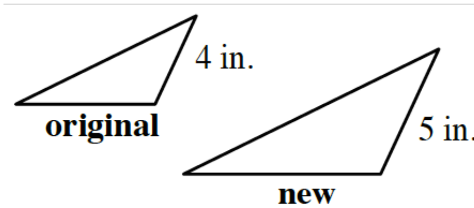
Problem 6-86

With your team, find the scale factor between each pair of similar shapes. That is, what are the sides of each original shape multiplied by to get the new shape?

a.



b.



Problem 6-87

It may have been easier to recognize the scale factor between the two shapes in part (a) of problem 6-86 than it was to determine the scale factor between the two shapes in part (b). When sides are not even multiples of each other (like the sides labeled 4 in. and 5 in. in part (b), it is useful to have another **strategy** for finding the scale factor.

Your Task: Work with your team to describe a **strategy** for finding the scale factor between any two shapes. Refer to the questions below to begin your discussion.

Discussion Points

How can we use pairs of corresponding sides to write the scale factor?

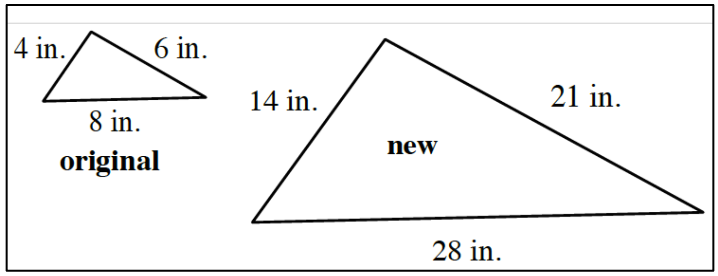
Will the scale factor between the shapes be more or less than 1?

Does it matter which pair of corresponding sides we use?

Problem 6-88

A study team was working together to find the scale factor for the two similar triangles below.

- Claudia set up the ratio $\frac{14}{4}$ to find the scale factor.
- Issac set up the ratio $\frac{28}{8}$ to find the scale factor.
- Paula set up the ratio $\frac{21}{6}$ to find the scale factor.



a. What did the students do differently when they found their scale factors?

b. Do the triangles have more than one scale factor? If not, show how they are the same.

c. Why does it make sense that the ratios are equal?

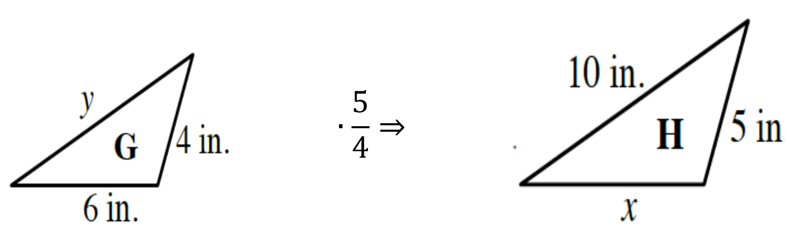
Problem 6-89

Alex was working with the two triangles from problem 6-86, but he now has a few more pieces of information about the sides. He has represented the new information and his scale factor in the diagram reprinted below.

a. Use the scale factor to find the length of the side labeled x. Show your work.

b. Since Alex multiplied the side lengths of triangle G to get triangle H, he needs to undo the enlargement to find the side labeled y. What math operation would he use to undo the enlargement? Write an expression and be prepared to explain your reasoning. If you are able, simplify the expression to find y.

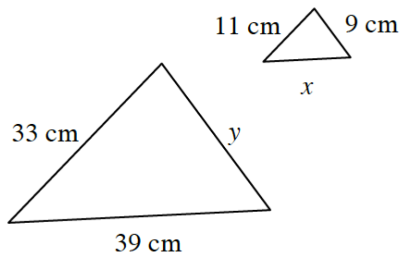
c. If triangle H had been the original triangle and triangle G had been the new triangle, how would the scale factor change? What would the new scale factor be? Explain.



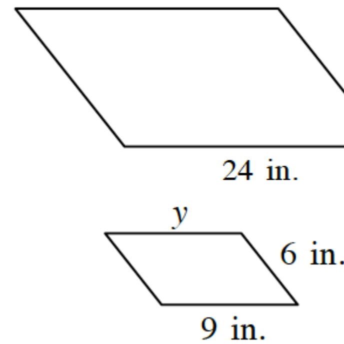
Problem 6-90

For the pairs of similar shapes below, find the lengths of the missing sides. Be sure to show your calculation. You can choose which shape is “new” and which is “original” in each pair. Assume the shapes are all drawn to scale. The shapes in part (b) are parallelograms and the shapes in part (d) are trapezoids.

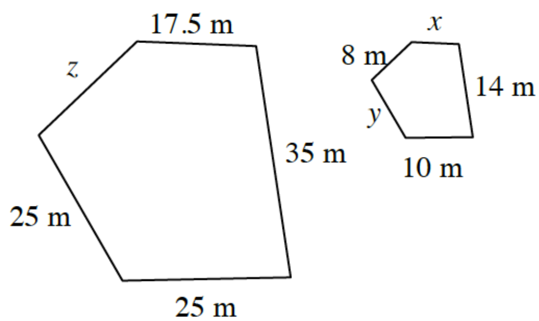
a.



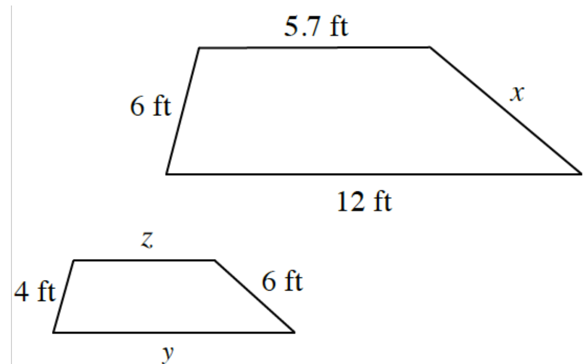
b.



c.

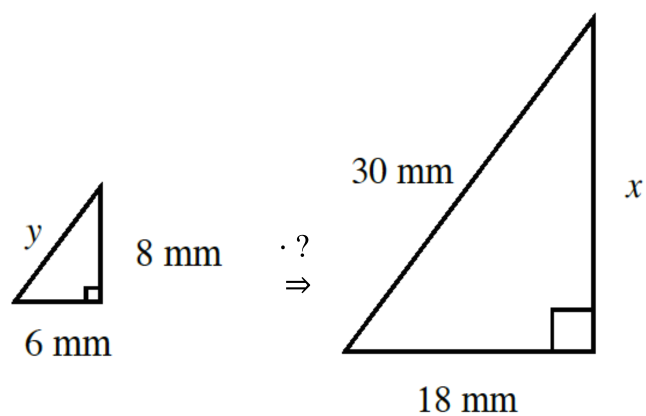


d.



Review & Preview — Problem 6-92

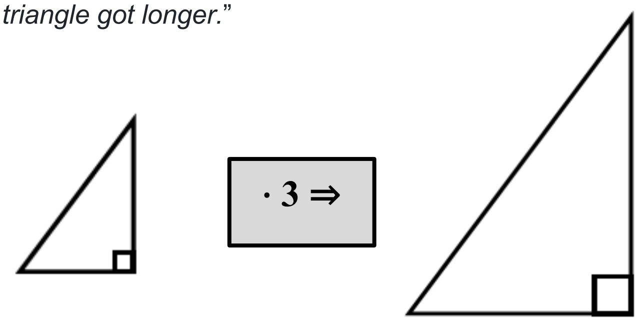
Find the scale factor of the two similar triangles and find the missing side lengths.



Problem 6-93

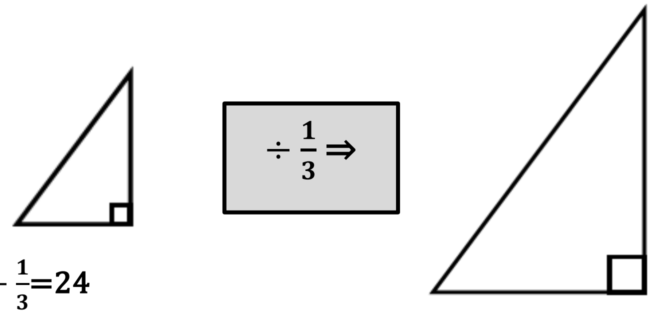
Alex and Maria were trying to find the side labeled x in problem 6-92. Their work is shown below.

Alex: "*I noticed that when I multiplied by* 3*, the sides of the triangle got longer.*"



$8 \cdot 3 = 24$

Maria: "*I remember that when we were dilating shapes in Lesson 6.2.2, my shape got bigger when I divided by* $\frac{1}{3}$."



$8 \div \frac{1}{3} = 24$

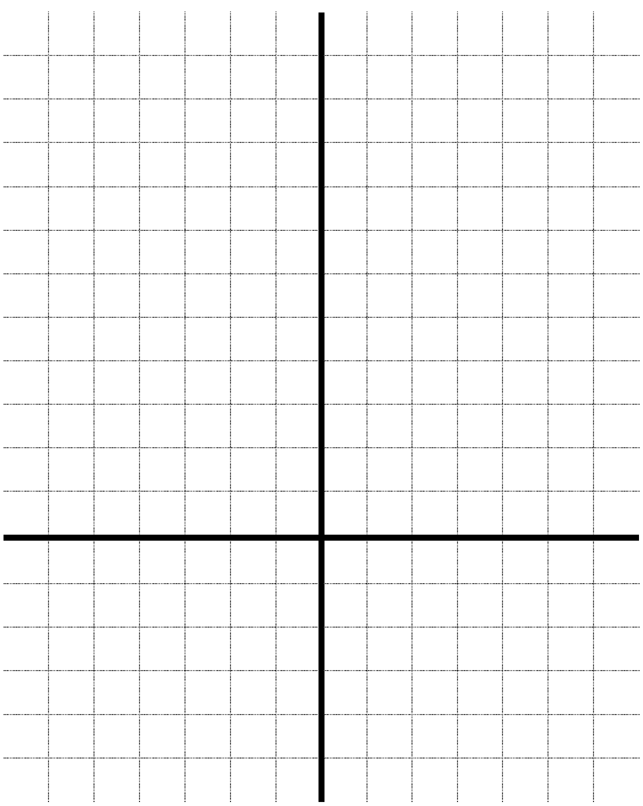
a. Look at each student's work. Why do both multiplying by 3 and dividing by $\frac{1}{3}$ make the triangles larger?

b. Use Alex and Maria's strategy to write two expressions to find the value of y in problem 6-92.

Problem 6-94

Consider these two equations: $y = 3x - 2$ $\quad$ $y = 4 + 3x$

a. Graph both equations on the same set of axes.



a. Solve this system using the Equal Values Method.

Explain how the answer to part (b) agrees with the graph you made in part (a).

Problem 6-95

Hollyhocks are tall, slender, flowering plants that grow in many areas of the U.S. Here are the heights (in inches) of hollyhocks that are growing in a park: **10, 39, 43, 45, 46, 47, 48, 48, 49, 50,** and **52.**

Find the median.

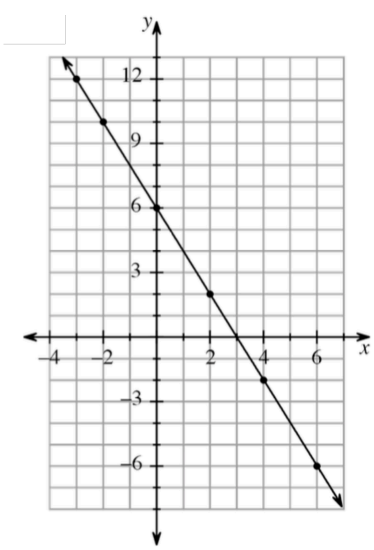
Find the quartiles.

Make a box plot of the data. (*Look at your toolkit to review how to build it.)*

Problem 6-96

Use the graph below to add points to the table below.

x	-3	-2	-1	0	1	2	3	4	5	6
y										



a. Write the rule in words.

b. Explain how to use the table to predict the value of y when x is −8.

Lesson 6.2.6
Solving Problems Involving Similar Shapes

How do I find a missing side?

Architects create scaled plans for building houses. Artists use sketches to plan murals for the sides of buildings. Companies create smaller sizes of their products to display in stores. Each of these models is created to show all of the information about the real object, without being the actual size of the object. Today you will work with your team to find strategies that you can use when you are missing some of the information about a set of similar shapes. As you work, look for more than one way to solve the problem.

Problem 6-98

MODEL TRAINS

Keenen loves trains, especially those that run on narrow-gauge tracks. (The gauge of a track measures how far apart the rails are.) He has decided to build a model train of the Rio Grande, a popular narrow-gauge train.

Use the following information to help him know how big his model should be:

- The real track has a gauge of 3 feet (36 inches).
- His model railroad track has a gauge of $\frac{3}{4}$ inches.
- The Rio Grande train he wants to model has driving wheels that measure 44 inches high.

Your Task: With your team, discuss what you know about the model train Keenen will build. What scale factor should he use? What will be the height of the driving wheels of his model? Be prepared to share your strategies with the class.

Problem 6-99

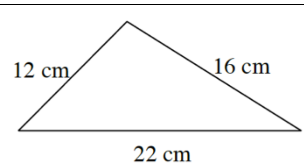
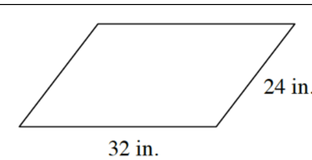
Heather and Cindy are playing "Guess My Shape." Heather has to describe a shape to Cindy accurately enough so that Cindy can draw it without ever seeing the shape. Heather gives Cindy these clues:

Clue #1: The shape is similar to a rectangle with a base of 7 cm and a height of 4 cm.

Clue #2: The shape is five times larger than the shape it is similar to.

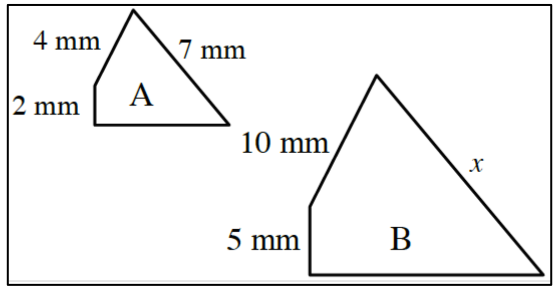
a. Has Heather given Cindy enough information to draw the shape? If so, sketch the shape on your paper. If not, write a question to ask Heather to get the additional information you need.

b. Use what you know about similar shapes to write a set of "Guess My Shape" clues to describe each of the mystery shapes below. Your clues should be complete enough for someone in another class to be able to draw the shape. Be sure to include at least one clue about the relationship between the mystery shape and a similar shape.

SHAPE A: Triangle	SHAPE B: Parallelogram
	

Problem 6-100

Nick enlarged figure A below so that it became the similar figure B. His diagrams are shown below.



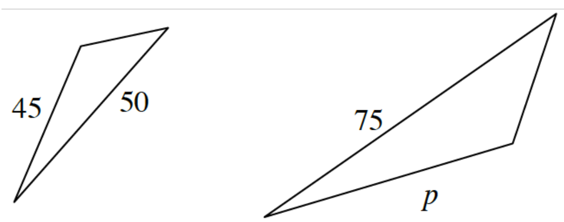
a. Write all of the ratios that compare the corresponding sides of figure B to figure A.

b. What is the relationship between these ratios? How do you know?

c. Use two different ways to find the value of x in this quadrilateral. Does your solution seem reasonable? Be ready to share your strategies with the class.

Problem 6-101

Fatima solved for p in the diagram of similar triangles below and got ***p=30.*** Looking at her answer, she knows she made a mistake. What would make Fatima think that her answer is wrong?



Problem 6-102

LEARNING LOG

In your Learning Log, write a description about how to find the missing side of a similar shape. Be specific about your strategy and include a picture with labels. Put today's date on your entry and title it "Finding Missing Sides of Similar Shapes."

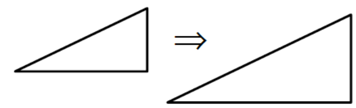
“Scale Factor”

Tool Kit page 44

Review & Preview — Problem 6-103

For each expression below:

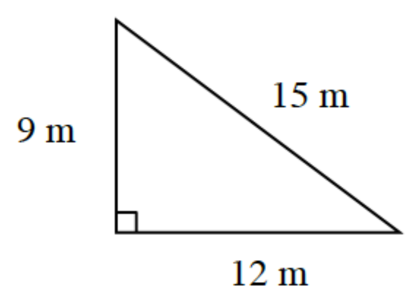
- Sketch and label a pair of similar shapes (like those at right or in problem 6-93) that would result in each calculation.
- Rewrite the expression so that the operation is multiplication.
- Calculate the value of the expression.



a. $6 \div \frac{1}{2}$	b. $4 \div \frac{2}{3}$

Problem 6-104

Draw a triangle with sides that are $\frac{1}{3}$ as long as the sides of the original triangle at right.



a. Calculate the perimeters of both triangles.

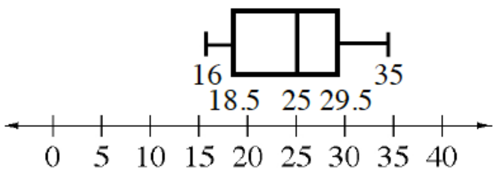
b. Calculate the areas of both triangles.

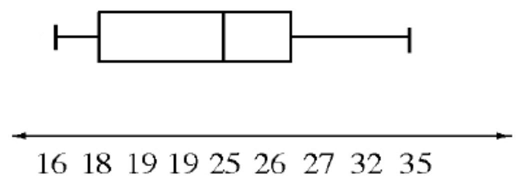
c. What is the relationship between the perimeters of the triangles?

Problem 6-105

Lucy and Marissa each designed a box plot to represent this data set: **16 18 19 19 25 26 27 32 35**

Their plots are shown below. Which plot is scaled correctly and why? Explain the mistakes in the incorrect plot.





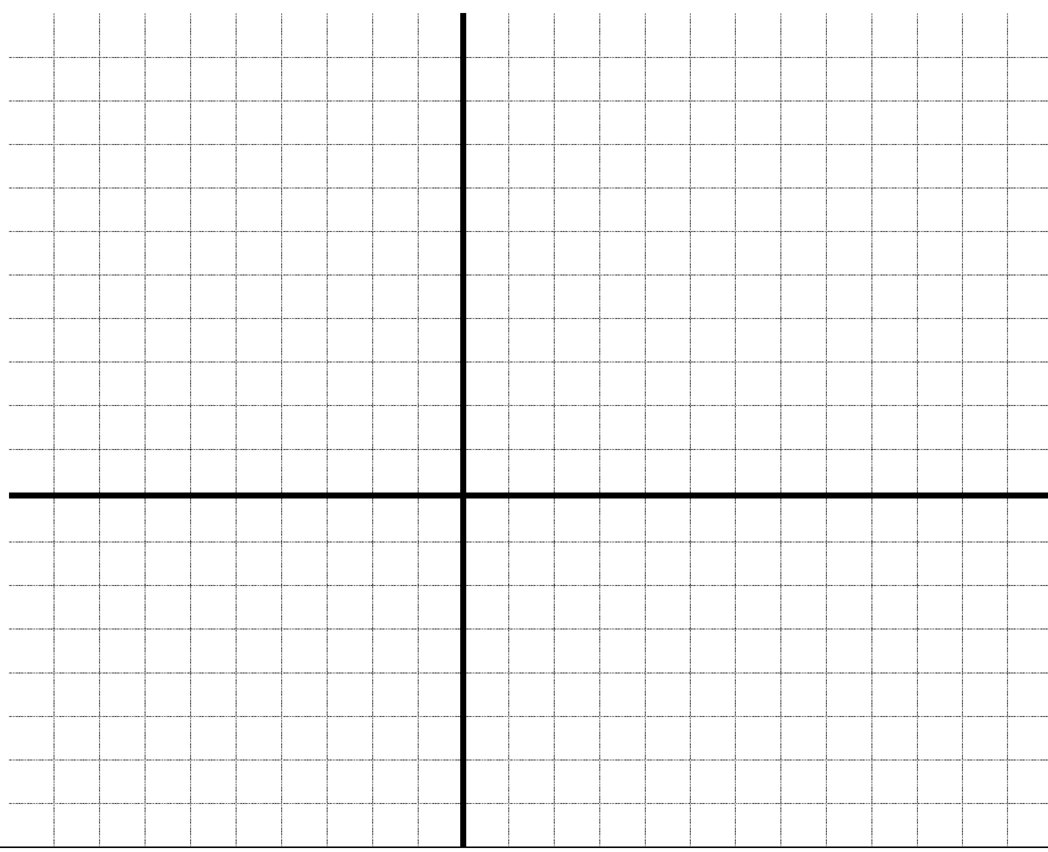
Problem 6-106

Plot and connect the points on the coordinate graph: $A(-3,1)$, $B(-1,3)$, $C(4,2)$, $D(2,0)$.

a. What is the shape you created?

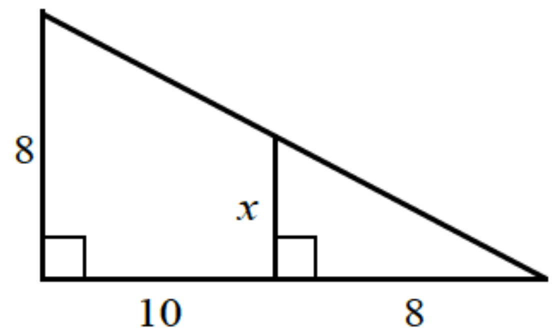
b. Reflect the shape across the x-axis. List the coordinates of the new points.

c. Multiply each coordinate of the original shape by 3. Graph the dilated shape.
What are the new coordinates of the points?



Problem 6-107

Examine the diagram below. The smaller triangle is similar to the larger triangle. Write and solve a proportion to find x. It may be helpful to draw the two triangles separately.



Problem 6-108

SEQUENCES OF TRANSFORMATIONS

a. A figure is rotated and reflected. What can you say about the new figure in relation to the original figure?

b. A figure is translated, reflected, and then dilated. What can you say about the new figure in relation to the original figure?

Chapter 6 Closure

CL 6-110

Priscilla and Ursula went fishing. Priscilla brought a full box of 32 worms and used one worm every minute. Ursula brought a box with five worms and decided to dig for more before she began fishing. Ursula dug up two worms per minute. When did Priscilla and Ursula have the same number of worms?

Define variables, write equations, and solve the system of equations.

CL 6-111

Use the graphs to complete each list of transformation steps you could use to move triangle B back to where it started at position A, and show each transformation on your graph.

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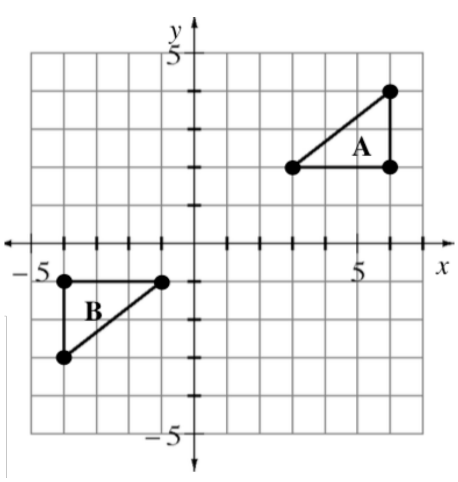
List I

1. Rotate triangle B 180° about point (−1,−1)
2. ?



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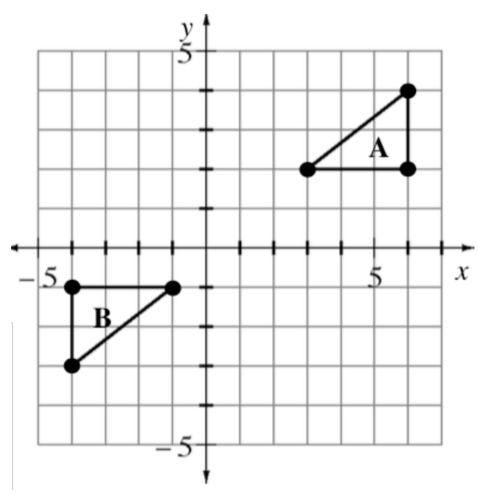
List II

1. Reflect triangle B across the y-axis.
2. ?
3. ?



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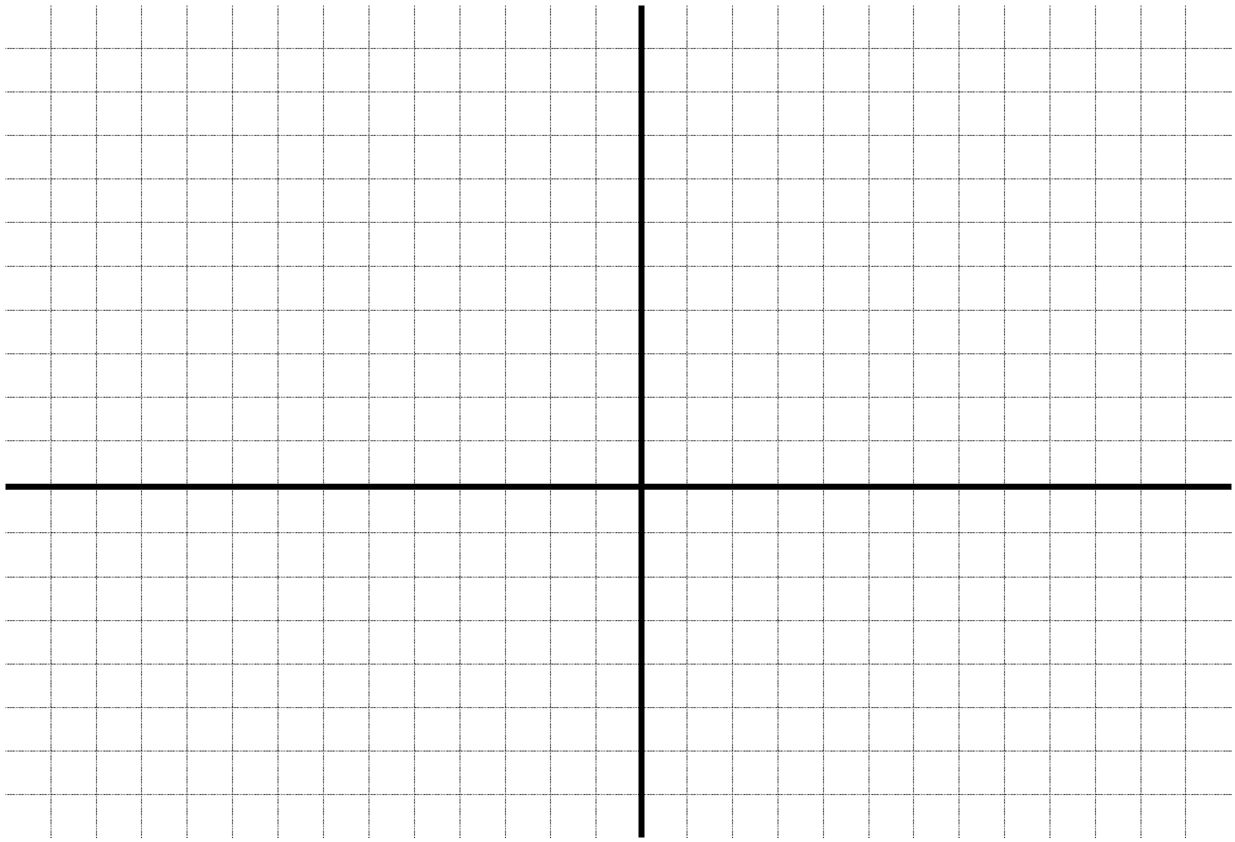
CL 6-112

Neatly graph the points (−2,9),(−3,7), and (−5,10) on a four-quadrant graph. Connect them to make a triangle. Then, for each transformation described below:

- Write and simplify an expression to find the new coordinates.
- Check your answer on your graph.

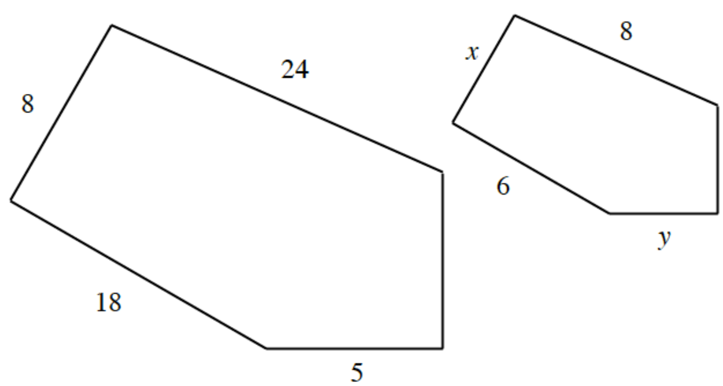
a. Slide the triangle right 4 units and down 6 units.
b. Reflect the triangle across the y-axis.



CL 6-113

The shapes at right are similar.

a. What is the scale factor?
b. Find the sides labeled x and y.



CL 6-114

As Khan and Jorman practice for college entrance tests, their scores increase. Khan's current score is 750 and is rising 8 points per week. Jorman's current score is 650 but is growing by 30 points per week. Write an equation or system of equations to determine when Jorman will catch up with Khan.

Define variables, write equations, and solve the system of equations.

CL 6-115

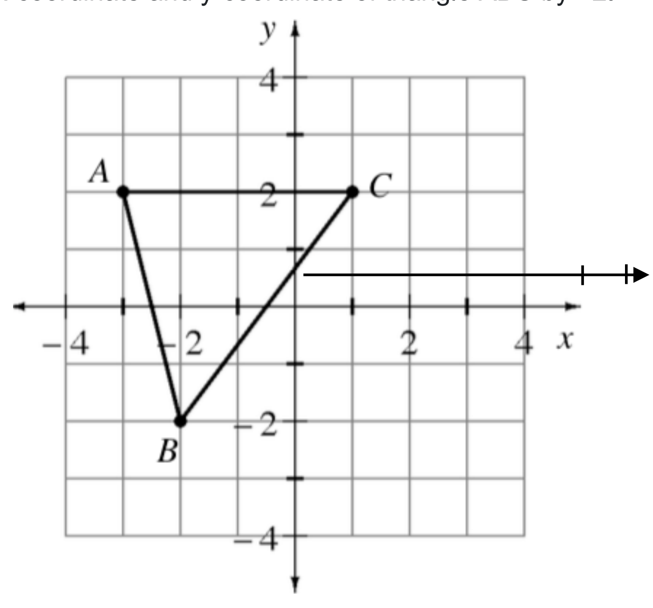
Solve each equation.

a. $\mathbf{3(2+x) = 4-(x-2)}$	b. $\mathbf{\frac{x}{2}+\frac{x}{3}-1=\frac{x}{6}+3}$

CL 6-116

Samantha is dilating triangle ABC at right. She multiplied each x-coordinate and y-coordinate of triangle ABC by −2.

a. Graph Samantha's new triangle.
b. Describe how triangle ABC changed.



CL 6-117

A trapezoid has a perimeter of 117 cm. The two shortest sides have the same length. The third side is 12 cm longer than one short side. The final side is 9 cm less than three times one short side. How long is each side of the trapezoid?

Define a variable and write an equation to represent this problem. Solve your equation and write your answer in a complete sentence.

Mathematical Vocabulary

The following is a list of vocabulary found in this chapter. Some of the words have been seen in a previous chapter. Make sure that you are familiar with the terms below and know what they mean. For more information, refer to the index. You might also add these words to your Toolkit so that you can reference them in the future.

congruent	**point of intersection**	**similar figures**
conjecture	**reduce**	**system of equations**
corresponding parts	**reflection**	**translation**
dilation	**rigid transformations**	**y-intercept**
enlarge	**rotation**	
linear equation	**scale factor**	